

MTH 351/651

Quiz #1

1. Consider the following ordinary differential equation

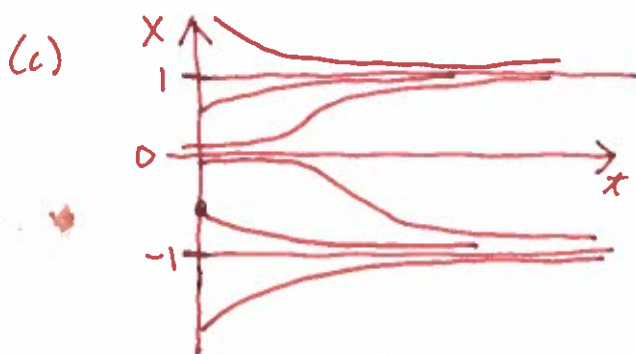
$$\dot{x} = a(x - x^3),$$

where $a > 0$ is a constant.

- Find all of the fixed points.
- Sketch the phase portrait and classify the stability of the fixed points.
- Sketch the graph of solutions $x(t)$ for different initial conditions. Be sure to include curves to illustrate all qualitatively different possibilities.
- Calculate \ddot{x} as a function of x .

(a) $x = 0, \pm 1$

(b) $\lim_{x \rightarrow \infty} a(x - x^3) = -\infty$



(d) $\ddot{x} = \frac{d}{dt} \dot{x} = \frac{d}{dx} a(x - x^3) \dot{x} = a^2(1 - 3x^2)(x - x^3).$