

MTH 317/617

Quiz #5

1. Show that if  $\xi$  is any value of

$$-i \log(i\sqrt{1-z^2} + z)$$

then

$$\cos(\xi) = z.$$

$$\begin{aligned} \cos(\xi) &= \frac{e^{i\xi} + e^{-i\xi}}{2} \\ &= \frac{e^{\log(i\sqrt{1-z^2} + z)} + e^{-\log(i\sqrt{1-z^2} + z)}}{2} \\ &= \frac{i\sqrt{1-z^2} + z + \frac{1}{i\sqrt{1-z^2} + z}}{2} \\ &= \frac{-(1-z^2) + 2(i\sqrt{1-z^2})z + z^2 + 1}{2(i\sqrt{1-z^2} + z)} \\ &= \frac{2z(z + i\sqrt{1-z^2})}{2(i\sqrt{1-z^2} + z)} \\ &= z. \end{aligned}$$