Quiz #5
MTH 351/651: Fall 2025
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Problem 1. Consider the following linear system of differential equations

$$\dot{\mathbf{x}} = A\mathbf{x}, \ \mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}, \ A = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}.$$

(a) Find the eigenvalues of A. **You do not have to find the eigenvectors.**

$\lambda_1 + \lambda_2 = 1$

$\lambda_1 \lambda_2 = 1$

$\Rightarrow \lambda_1 = (1-\lambda_2)$

$\Rightarrow (1-\lambda_2)\lambda_2 = 1$

$\lambda_2^2 - \lambda_2 + 1$

$\Rightarrow \lambda_2 = \frac{1}{2} \pm \frac{\sqrt{1-4}}{2}$

$= \frac{1}{2} \pm \frac{\sqrt{3}}{2} i$ (unstable spiral)

(b) Determine the null clines of this system.

$\dot{x} = 0 \Rightarrow y = 0$

$\dot{y} = 0 \Rightarrow y = x$

(c) On the axes below, sketch a phase portrait of this system. Your phase portrait must be consistent with the null clines you found as well as the overall directions of the underlying vectorfield.