

MTH 113
Fall 2026
Exam 2
10/16/26

Name (Print):

Key

This exam contains 7 pages (including this cover page) and 8 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

The following rules apply:

- If you use a “fundamental theorem” you must indicate this and explain why the theorem may be applied.
- Organize your work, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- Short answer questions: Questions labeled as “Short Answer” can be answered by simply writing an equation or a sentence or appropriately drawing a figure. No calculations are necessary or expected for these problems.
- Unless the question is specified as short answer, mysterious or unsupported answers might not receive full credit. An incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- Clearly justify your answers to calculation and proof questions. For Correct/Incorrect and Short Answer questions, follow the specific instructions given.

Problem	Points	Score
1	15	
2	15	
3	10	
4	15	
5	10	
6	15	
7	0	
8	10	
Total:	90	

Do not write in the table to the right.

1. (15 points) **Correct or Incorrect (Short Answer).** Circle **C** or **I** for each statement. A statement is correct only if it is true in every case. No explanation is necessary.

(a) **C** **I** If $\vec{u}$ and $\vec{v}$ are vectors in $\mathbb{R}^3$ satisfying $\vec{u} \cdot \vec{v} = 0$, then $\vec{u} \times \vec{v} = \vec{0}$.

(b) **C** **I** If $\vec{u}$ and $\vec{v}$ are vectors in $\mathbb{R}^3$ satisfying $\vec{u} \cdot \vec{v} = 0$ and $\vec{u} \times \vec{v} = \vec{0}$, then $\vec{u} = \vec{0}$ or $\vec{v} = \vec{0}$.

(c) **C** **I** If $\vec{a}$ and $\vec{b}$ are vectors in $\mathbb{R}^3$ satisfying $\|\vec{a}\| = \|\vec{b}\|$, then $\vec{a} = \vec{b}$.

(d) **C** **I** If $\lim_{(x,y) \rightarrow (0,0)} f(x,y) = 5$, then $\lim_{x \rightarrow 0} f(x,0) = 5$.

(e) **C** **I** If $\lim_{x \rightarrow 0} f(x,0) = 5$, then $\lim_{(x,y) \rightarrow (0,0)} f(x,y) = 5$.

2. (15 points) Consider the surface $z = f(x,y) = \frac{1}{2}x^2 + 4y^2$. Find an equation of the tangent plane to the surface at $(2,1,6)$.

$$\frac{\partial f}{\partial x} = x, \quad \frac{\partial f}{\partial y} = 8y$$

$$\Rightarrow \left. \frac{\partial f}{\partial x} \right|_{(2,1)} = 2, \quad \left. \frac{\partial f}{\partial y} \right|_{(2,1)} = 8$$

$$\Rightarrow L(x,y) = 2(x-2) + 8(y-1) + 6$$

3. (10 points) **Short Answer:** Use the following contour plots to answer the following questions. To answer each question you can simply write down the letter of the corresponding contour plot.

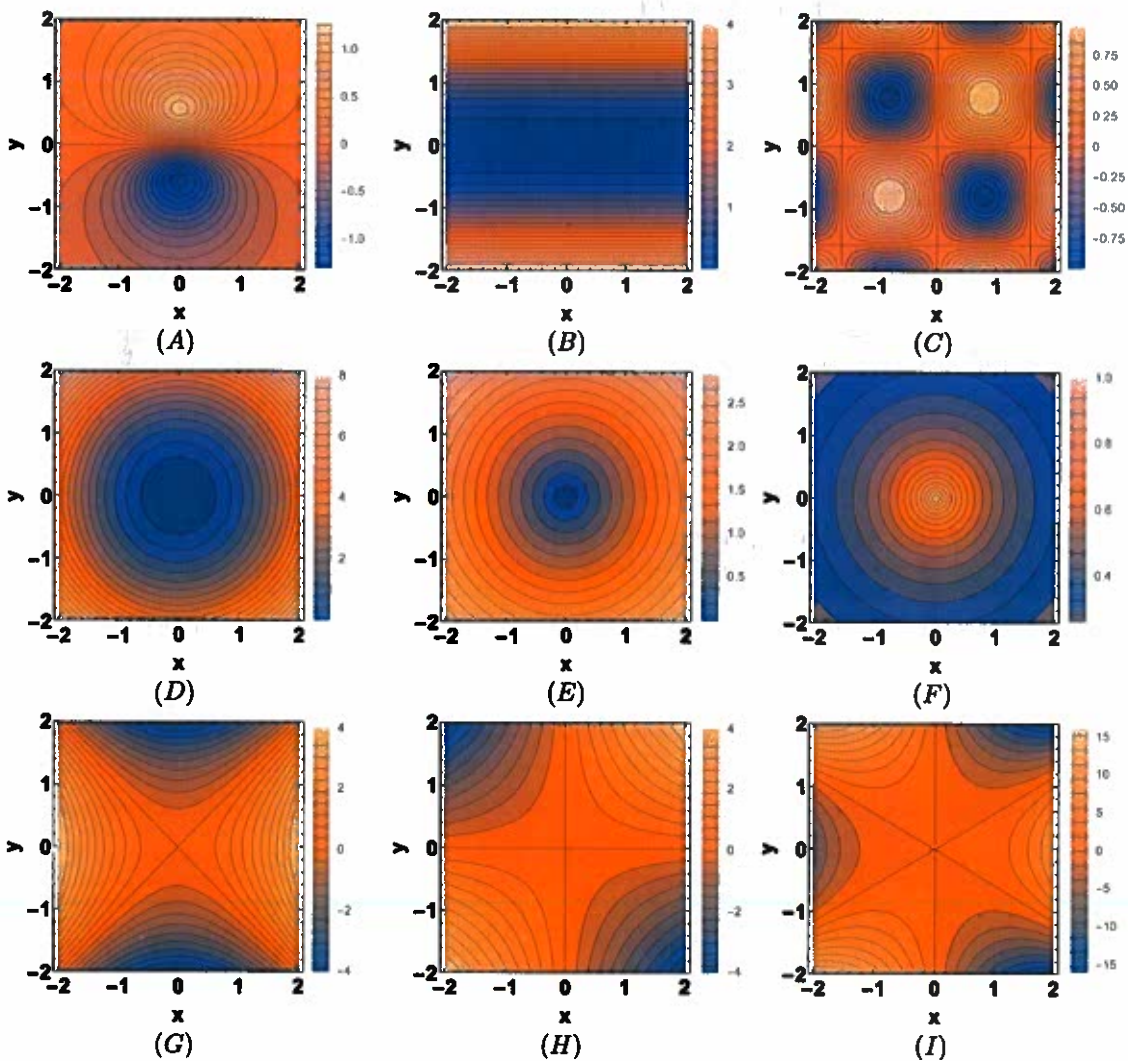
(a) (2 points) Which picture represents the level curves of $f(x, y) = y^2$? **B**

(b) (2 points) Which picture represents the level curves of $f(x, y) = \sin(2x) \sin(2y)$? **C**

(c) (2 points) Which picture represents the level curves of $f(x, y) = xy$? **H**

(d) (2 points) Which picture represents the level curves of $f(x, y) = \sqrt{x^2 + y^2}$? **E**

(e) (2 points) Which picture represents the level curves of $f(x, y) = x^2 + y^2$? **D**



4. (15 points) Let $A = (1, 0, 2)$, $B = (3, 1, 0)$, and $C = (0, 2, 1)$.

(a) (5 points) Compute $\vec{AB}$ and $\vec{AC}$.

$$\vec{AB} = \langle 2, 1, -2 \rangle$$

$$\vec{AC} = \langle -1, 2, -1 \rangle$$

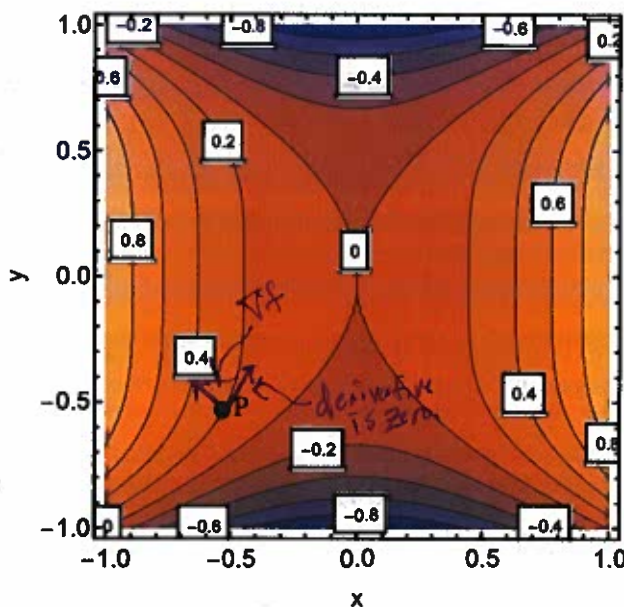
(b) (5 points) Compute $\vec{AB} \times \vec{AC}$.

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & -2 \\ -1 & 2 & -1 \end{vmatrix} = 3\vec{i} + 4\vec{j} + 5\vec{k}$$

(c) (5 points) Find an equation of the plane through the points A , B , and C .

$$3(x-1) + 4y + 5(z-2) = 0$$

5. (10 points) A contour plot for the differentiable function $f(x, y)$ is given below.



- (a) (2.5 points) (Short Answer:) At the point P is f_x positive or negative?

Negative

- (b) (2.5 points) (Short Answer:) At the point P is f_y positive or negative?

Negative

- (c) (2.5 points) (Short Answer:) On the contour plot, sketch the direction of ∇f at P .

- (d) (2.5 points) (Short Answer:) On the contour plot, sketch a direction at P in which the directional derivative of f is zero.

6. (15 points) Suppose $z = f(x, y)$, where $x = r \cos \theta$ and $y = r \sin \theta$.

(a) (10 points) Use the chain rule to derive an expression for $\frac{\partial z}{\partial \theta}$ in terms of $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, r , and θ .

$$\frac{\partial z}{\partial \theta} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial \theta} = -r \sin \theta \frac{\partial f}{\partial x} + r \cos \theta \frac{\partial f}{\partial y}$$

(b) (5 points) If $\frac{\partial f}{\partial x} = 3$ and $\frac{\partial f}{\partial y} = -1$ when $x = 0$ and $y = 2$, use your result from part (a) to find $\frac{\partial z}{\partial \theta}$ when $r = 2$ and $\theta = \frac{\pi}{2}$.

$$\left. \frac{\partial z}{\partial \theta} \right|_{(r=2, \theta=\pi/2)} = -2 \sin(\pi/2) \cdot 3 + 2 \cos(\pi/2) (-1) = -6.$$

7. (10 points) Consider the function $f(x, y) = x^2y + 3y^2$.

(a) (5 points) Compute $\nabla f(1, 1)$.

$$\nabla f = \langle 2xy, x^2 + 6y \rangle$$

$$\Rightarrow \nabla f|_{(1,1)} = \langle 2, 7 \rangle$$

(b) (5 points) Find a unit vector $\vec{u}$ such that the directional derivative of f at $(1, 1)$ in the direction $\vec{u}$ is maximized.

$$\vec{u} = \frac{\langle 2, 7 \rangle}{\sqrt{53}}$$

8. (10 points) Determine whether the following limit exists. If it exists, use the squeeze theorem to justify its value; if it does not, justify your conclusion by considering appropriate paths:

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^2}{x^4 + y^4}.$$

$$\lim_{(x,0) \rightarrow (0,0)} \frac{x^2 \cancel{y^2}}{x^4 + \cancel{y^4}} = \lim_{x \rightarrow 0} \frac{0}{x^4} = 0$$

$$\lim_{(x,x) \rightarrow (0,0)} \frac{x^2 \cancel{y^2}}{x^4 + \cancel{y^4}} = \lim_{x \rightarrow 0} \frac{x^4}{2x^4} = 1/2.$$

The limit does not exist.