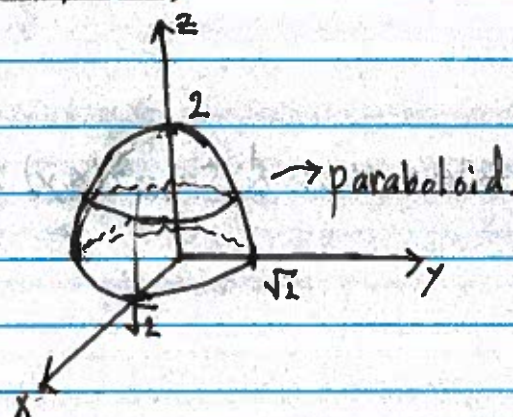


Homework #5

pg. 947, #28



pg. 948, #32

(a) III

(b) I

(c) IV

(d) V

(e) VI

(f) II

pg. 950, #61-66

#61: C, II (Function is zero along the lines $x=0$ and $y=0$ and oscillates)

#62: A, IV (Function diverges in x -direction and oscillates in y direction)

#63: F, I (Function is constant along lines $y=x+k$)

#64: E, III (Function is 2π periodic for fixed x or y)

#65: B, VI (Function vanishes along $x=\pm 1$ and $y=\pm 1$)

#66: D, V (Function vanishes along $y=x$)

pg. 960, #14

$$\lim_{(x,y) \rightarrow (0,0)} \frac{2xy}{x^2+y^2} = \lim_{x \rightarrow 0} \frac{2x^2}{4x^2} = \lim_{x \rightarrow 0} \frac{1}{2} = \frac{1}{2}.$$

$$\lim_{(x,0) \rightarrow (0,0)} \frac{2xy}{x^2+y^2} = \lim_{x \rightarrow 0} \frac{0}{x^2} = 0.$$

Since $\lim_{(x,y) \rightarrow (0,0)} f(x,y) \neq \lim_{(x,0) \rightarrow (0,0)} f(x,y)$ it follows that $\lim_{(x,y) \rightarrow (0,0)} f(x,y)$ does not exist.

pg. 960, #22

$$\lim_{(x,y) \rightarrow (1,2)} \frac{2x-y}{4x^2-y^2} = \lim_{(x,y) \rightarrow (1,2)} \frac{2x-y}{(2x-y)(2x+y)} = \lim_{(x,y) \rightarrow (1,2)} \frac{1}{2x+y} = \frac{1}{4}.$$

pg. 960, #26

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy^4}{x^2+y^2} = \lim_{x \rightarrow 0} \frac{x^5}{x^2+x^2} = \lim_{x \rightarrow 0} \frac{x^3}{1+x^2} = 0.$$

$$\lim_{(x,0) \rightarrow (0,0)} \frac{xy^4}{x^2+y^2} = \lim_{x \rightarrow 0} \frac{0}{x^2} = 0.$$

Since $\lim_{(x,y) \rightarrow (0,0)} f(x,y) \neq \lim_{(x,0) \rightarrow (0,0)} f(x,y)$ it follows that $\lim_{(x,y) \rightarrow (0,0)} f(x,y)$ does not exist.

pg. 960, #32

$$\left| \frac{xy}{\sqrt{x^2+y^2}} \right| = \frac{|x||y|}{\sqrt{x^2+y^2}} \leq \frac{|x||y|}{\sqrt{x^2}} = \frac{|x||y|}{|x|} = |y|$$

Since $\lim_{(x,y) \rightarrow (0,0)} |y| = 0$ it follows from the squeeze theorem that

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{\sqrt{x^2+y^2}} = 0.$$

pp. 970, #16

$$g(x, y) = (x^2 + xy)^3$$

$$\frac{\partial g}{\partial x} = 3(x^2 + xy)^2(2x + y)$$

$$\frac{\partial g}{\partial y} = 3(x^2 + xy)^2 x$$

pp. 970, #24

$$f(x, y) = x^y$$

$$\frac{\partial f}{\partial x} = y x^{y-1}$$

$$\ln(f) = y \ln(x)$$

$$\Rightarrow \frac{\partial}{\partial y} \ln(f) = \frac{\partial}{\partial y} (y \ln(x))$$

$$\Rightarrow \frac{1}{f} \frac{\partial f}{\partial y} = \ln(x)$$

$$\Rightarrow \frac{\partial f}{\partial y} = f \cdot \ln(x)$$

$$\Rightarrow \frac{\partial f}{\partial y} = \ln(x) x^y$$

pp. 970, #26

$$F(\alpha, \beta) = \int_{\alpha}^{\beta} \sqrt{t^3 + 1} dt$$

$$\frac{\partial F}{\partial \alpha} = -\sqrt{\alpha^3 + 1}$$

$$\frac{\partial F}{\partial \beta} = \sqrt{\beta^3 + 1}$$