

## Homework #6

pg. 970, #46

$$(a) z = f(x)g(y)$$

$$\Rightarrow \frac{\partial z}{\partial x} = \frac{\partial f}{\partial x} g, \quad \frac{\partial z}{\partial y} = f(x) \frac{\partial g}{\partial y}$$

$$(b) z = f(xy)$$

$$\Rightarrow \frac{\partial z}{\partial x} = y f'(xy), \quad \frac{\partial z}{\partial y} = x f'(xy)$$

$$(c) z = f\left(\frac{x}{y}\right)$$

$$\Rightarrow \frac{\partial z}{\partial x} = \frac{1}{y} f'\left(\frac{x}{y}\right), \quad \frac{\partial z}{\partial y} = -\frac{x}{y^2} f'\left(\frac{x}{y}\right)$$

pg. 970, #68

$$g(x, y, z) = \sqrt{1+xz} + \sqrt{1-xy}$$

$$g_{xyz} = \frac{\partial^3}{\partial x \partial y \partial z} \left( \sqrt{1+xz} + \sqrt{1-xy} \right)$$

$$= \frac{\partial^3}{\partial x \partial y \partial z} \left( \sqrt{1+xz} \right) + \frac{\partial^3}{\partial x \partial y \partial z} \left( \sqrt{1-xy} \right)$$

$$= \frac{\partial^3}{\partial x \partial z \partial y} \left( \sqrt{1+xz} \right) + 0$$

$$= 0$$

pg. 981, #10

$$z = L(x-2y)$$

$$\left. \frac{\partial z}{\partial x} \right|_{(3,1)} = \frac{1}{x-2y} \Big|_{(3,1)} = 1$$

$$\Rightarrow L(x, y) = (x-3) - 2(y-1)$$

$$\left. \frac{\partial z}{\partial y} \right|_{(3,1)} = \frac{-2}{x-2y} \Big|_{(3,1)} = -2$$

pg. 982, #24

$$z = \frac{y-1}{x+1}$$

$$\Rightarrow \left. \frac{\partial z}{\partial x} \right|_{(0,0)} = \left. \frac{-(y-1)}{(x+1)^2} \right|_{(0,0)} = 1, \quad \left. \frac{\partial z}{\partial y} \right|_{(0,0)} = \left. \frac{1}{x+1} \right|_{(0,0)} = 1, \quad z|_{(0,0)} = -1$$

Therefore, near  $(0,0)$

$$\frac{y-1}{x+1} \approx x+y-1.$$

pg. 982, #40

$$z = x^2 - xy + 3y^2$$

$$\frac{\partial z}{\partial x} = 2x - y, \quad \frac{\partial z}{\partial y} = -x + 6y$$

$$\Rightarrow \left. \frac{\partial z}{\partial x} \right|_{(3,-1)} = 7, \quad \left. \frac{\partial z}{\partial y} \right|_{(3,-1)} = -9$$

$$\Rightarrow \Delta z = 7\Delta x + 9\Delta y$$

$$\Delta z = 7 \cdot (-.04) + 9 \cdot (.05) = -.28 + .45 = .17$$

pg. 991, #6

$$z = \sqrt{1+xy}, \quad x = \tan(t), \quad y = \arctan(t)$$

$$\Rightarrow \frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt}$$

$$= \frac{y(1+xy)^{-1/2} \sec^2(t)}{2} + \frac{x(1+xy)^{-1/2}}{2} \cdot \frac{1}{1+t^2} \quad \leftarrow \text{This is fine.}$$

$$= \frac{(1+xy)^{-1/2}}{2} \left( \arctan(x) \sec^2(t) + \frac{\tan(t)}{1+t^2} \right)$$

$$= \frac{(1+\tan(t)\arctan(t))^{-1/2}}{2} \left( \arctan(t) \sec^2(t) + \frac{\tan(t)}{1+t^2} \right)$$

pg 991, #16

$$z = \tan(u/v), u = 2s + 3t, v = 3s - 2t$$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial u} \frac{\partial u}{\partial s} + \frac{\partial z}{\partial v} \frac{\partial v}{\partial s}$$

$$= \frac{1}{v} \sec^2\left(\frac{u}{v}\right) \cdot 2 + \frac{u}{v^2} \sec^2\left(\frac{u}{v}\right) \cdot 3$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial u} \frac{\partial u}{\partial t} + \frac{\partial z}{\partial v} \frac{\partial v}{\partial t}$$

$$= \frac{1}{v} \sec^2\left(\frac{u}{v}\right) \cdot 3 + \frac{u}{v^2} \sec^2\left(\frac{u}{v}\right) (-2)$$

pg 992, #40

(a)  $\frac{\partial W}{\partial T} = -2 \Rightarrow$  wheat production goes down with rising temperatures.

(b)  $\frac{\partial W}{\partial R} = 8 \Rightarrow$  wheat production goes up with rising rainfall.

$$(b) \frac{dW}{dt} = \frac{\partial W}{\partial T} \frac{\partial T}{\partial t} + \frac{\partial W}{\partial R} \frac{\partial R}{\partial t} \approx (-2)(1.5) + 8 \cdot (-.1) = -1.1$$

pg 1006, #34

$$f(x, y) = x^2 + xy^3$$

$$\nabla f = \langle 2x + y^3, 3xy^2 \rangle$$

$$\nabla f(2, 1) = \langle 5, 6 \rangle$$

If  $\vec{U} = \langle u_1, u_2 \rangle$  is a unit vector then

$$\nabla f(1, 2) \cdot \langle u_1, u_2 \rangle = \|\vec{U}\| \cdot \|\nabla f\| \cos \theta = 2$$

$$\Rightarrow \cos \theta = \frac{2}{\sqrt{61}} \Rightarrow \vec{U} = \left\langle \frac{2}{\sqrt{61}}, \frac{57}{\sqrt{61}} \right\rangle$$

pg. 1008, #40

$$z = 1000 - .005x^2 - .01y^2$$

$$\frac{\partial z}{\partial x} = -.01x \Rightarrow \left. \frac{\partial z}{\partial x} \right|_{(60, 40)} = -.6$$

$$\frac{\partial z}{\partial y} = -.02y \Rightarrow \left. \frac{\partial z}{\partial y} \right|_{(60, 40)} = -.8$$

(a) Ascend

$$(b) \nabla f(60, 40) \cdot \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = .6/\sqrt{2} - .8/\sqrt{2} < 0$$

Descend at rate  $\cdot 2/\sqrt{2}$

$$(c) \nabla f(60, 40) = \langle -.6, -.8 \rangle$$

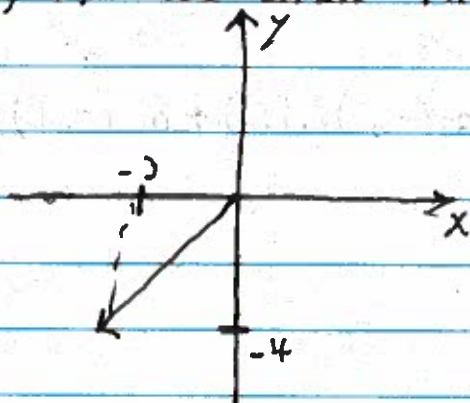
The direction of greatest ascent is

$$\vec{u} = \frac{\langle -3, -4 \rangle}{5} = \left\langle \frac{-3}{5}, \frac{-4}{5} \right\rangle$$

The rate of change of  $f$  in this direction is

$$\|\nabla f\| = \sqrt{.6^2 + .8^2} = \sqrt{.36 + .64} = 1.$$

Finally, if we draw the direction  $\vec{u}$  we have that



The angle with the  $x$ -axis is then  $\pi + \cos^{-1}(\frac{3}{5})$ .