

Homework #7

pg. 1017, #10

$$f(x, y) = y(e^x - 1)$$

$$\nabla f = \langle ye^x, e^x - 1 \rangle$$

$$\nabla f = 0 \Rightarrow x = 0, y = 0$$

We also have

$$\frac{\partial^2 f}{\partial x^2} = ye^x, \quad \frac{\partial^2 f}{\partial x \partial y} = e^x, \quad \frac{\partial^2 f}{\partial y^2} = 0$$

$$\Rightarrow D|_{(0,0)} = 0 \cdot 0 - 1 = -1$$

Consequently, $(0, 0)$ is a saddle.

pg. 1017, #18

$$f(x, y) = (6x - x^2)(4y - y^2)$$

$$\nabla f = \langle (6 - 2x)(4y - y^2), (6x - x^2)(4 - 2y) \rangle$$

$$\nabla f = 0 \Rightarrow x = 3, y = 2 \text{ or } x = 0, y = 0 \text{ or } x = 0, y = 2 \text{ or } x = 6, y = 0 \text{ or } x = 6, y = 2$$

We also have

$$\frac{\partial^2 f}{\partial x^2} = -2(4y - y^2), \quad \frac{\partial^2 f}{\partial x \partial y} = (6 - 2x)(4 - 2y), \quad \frac{\partial^2 f}{\partial y^2} = -2(6x - x^2)$$

$$\Rightarrow D|_{(0,0)} = 0 \cdot 0 - (6 \cdot 4) < 0$$

$$D|_{(0,2)} = 0 - [(6)(-4)] < 0$$

$$D|_{(6,0)} = 0 - (-6)(4) < 0$$

$$D|_{(3,2)} = (-2)(4)(-2)(18-9) > 0, \quad \frac{\partial^2 f}{\partial x^2}|_{(3,2)} = -2(4) < 0$$

$$D|_{(6,2)} = (-2)(3)(-2)(0) - [(-6)(-2)]^2 < 0$$

$(3, 2)$ is a local min

$(0, 0), (0, 4), (6, 0), (6, 4)$ are saddles.

pg. 1018, #46

The square distance is given by

$$S = x^2 + y^2 + z^2 \\ = x^2 + 9 + xz + z^2$$

$$\Rightarrow \nabla S = \langle 2x+z, x+2z \rangle$$

Critical Points satisfy

$$2x+z=0 \quad x+2z=0$$

Therefore, $(0,0)$ is the only critical point. This clearly minimizes distance and the **points** on the graph are $(0,3,0)$ and $(0,0,3)$.

pg. 1018, #54

Let C be the cost of glass and V the total volume. If the dimensions are L, W , and H then the total cost is

$$T = 5C \cdot L \cdot W + 2C \cdot L \cdot H + 2C \cdot W \cdot H$$

Since $V = L \cdot W \cdot H$ we have that $H = \frac{V}{L \cdot W}$. Consequently,

$$T = 5CLW + \frac{2CV}{W} + \frac{2CV}{L}$$

$$\frac{\partial T}{\partial L} = 5CW - \frac{2CV}{L^2}, \quad \frac{\partial T}{\partial W} = 5CL - \frac{2CV}{W^2}$$

The critical point will satisfy $L=W$ and thus

$$\frac{5CW - 2CV}{W^2} = 0 \Rightarrow W^3 = \frac{2}{5}V \Rightarrow W = \left(\frac{2V}{5}\right)^{1/3}$$

The dimensions are thus

$$L = \left(\frac{2V}{5}\right)^{1/3}$$

$$W = \left(\frac{2V}{5}\right)^{1/3}$$

$$H = \frac{V}{\left(\frac{2V}{5}\right)^{2/3}} = \left(\frac{5}{2}\right)^{2/3} V^{1/3}$$

$$\frac{5}{2} \left(\frac{2V}{5}\right)^{1/3}$$

pg. 1018, #60.

$$P = 2pq + 2pr + 2rq$$

$$p + q + r = 1 \Rightarrow r = 1 - p - q$$

$$\Rightarrow P = 2pq + 2p(1 - p - q) + 2(1 - p - q)q$$

$$\Rightarrow P = 2pq + 2p - 2p^2 - 2pq + 2q - 2pq - 2q^2$$

$$\Rightarrow P = 2(p - p^2 + q - q^2 - pq)$$

$$\nabla P = 2 \langle 1 - 2p - q, 1 - 2q - p \rangle$$

Clearly critical points satisfy $p = q$ and thus

$$1 - 3p = 0$$

$$\Rightarrow p = 1/3$$

The critical point is then $(1/3, 1/3, 1/3)$.

$$P(1/3, 1/3, 1/3) = (2/9 + 1/9 + 1/9) = 6/9 = 2/3$$