

Problem 1. Suppose X, Y are independent, identically distributed random variables with the following densities:

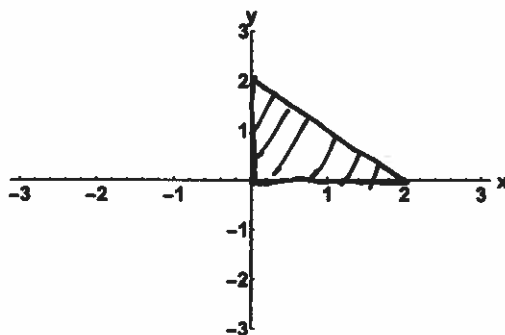
$$f(x) \sim \begin{cases} e^{-x} & x > 0 \\ 0 & x \leq 0 \end{cases} \text{ and } g(y) \sim \begin{cases} e^{-y} & y > 0 \\ 0 & y \leq 0 \end{cases},$$

respectively.

(a) Write down the joint probability density of $\vec{X} = (X, Y)$.

$$F(x, y) = \begin{cases} e^{-x}e^{-y}, & x > 0, y > 0 \\ 0, & \text{o.w.} \end{cases}$$

(b) On the axis below, sketch the region satisfying: $x + y < 2$, $x > 0$, and $y > 0$.



(c) Compute $P(X + Y < 2)$.

$$\begin{aligned} P(X + Y < 2) &= P(Y < 2 - X) \\ &= \int_0^2 \int_0^{2-x} e^{-x} e^{-y} dy dx \\ &= \int_0^2 e^{-x} (1 - e^{-2+x}) dx \\ &= \int_0^2 (e^{-x} - e^{-2}) dx \\ &= 1 - e^{-2} - 2e^{-2} \\ &= 1 - 3e^{-2} \end{aligned}$$

Quiz #3G

MTH 383/683: Fall 2026

Instructor: John Gemmer

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$$f(x) \sim \begin{cases} e^{-x} & x > 0 \\ 0 & x \leq 0 \end{cases} \text{ and } g(y) \sim \begin{cases} e^{-y} & y > 0 \\ 0 & y \leq 0 \end{cases},$$

respectively. Compute $P(X + Y < 2)$.

The joint density is

$$F(x, y) = \begin{cases} e^{-x}e^{-y}, & x > 0, y > 0 \\ 0, & \text{o.w.} \end{cases}$$

Therefore,

$$\begin{aligned} P(X+Y < 2) &= P(Y < 2-X) \\ &= \int_0^2 \int_0^{2-x} e^{-x}e^{-y} dy dx \\ &= \int_0^2 e^{-x}(1-e^{-2+x}) dx \\ &= \int_0^2 (e^{-x} - e^{-2}) dx \\ &= 1 - e^{-2} - 2e^{-2} \\ &= 1 - 3e^{-2} \end{aligned}$$