Homework #10

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$$\sum_{n=1}^{\infty} \frac{2^n+4^n}{e^n} = \sum_{n=1}^{\infty}\left(\frac{2}{e}\right)^n + \sum_{n=1}^{\infty}\left(\frac{4}{e}\right)^n.$$

Therefore,

$$\sum_{n=1}^{\infty} \frac{2^n+4^n}{e^n}$$

is divergent.

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$$\sum_{n=1}^{\infty} (x+2)^n = \frac{1}{1-(x+2)}, \text{ if } |x+2|<1.$$

Therefore,

$$\sum_{n=1}^{\infty} (x+2)^n = \frac{-1}{1+x}, \text{ if } -3<x<-1.$$

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$$\sum_{n=0}^{\infty} e^{nc} = 10$$

$$\Rightarrow \sum_{n=0}^{\infty} (e^c)^n = 10$$

$$\Rightarrow \frac{1}{1-e^c} = 10$$

$$\Rightarrow 1-e^c = \frac{1}{10}$$

$$\Rightarrow c = \ln(9/10)$$

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$$\sum_{k=1}^{\infty} ke^{-k^2}.$$

Let $f(x)=xe^{-x^2}$. Calculating,

$$f'(x)=e^{-x^2}-2x^2e^{-x^2}=(1-2x^2)e^{-x^2}.$$

Therefore, $f(x)$ is decreasing for $x>1/\sqrt{2}$. Applying the integral test:

$$\int_1^{\infty} xe^{-x^2}dx=\frac{1}{2}\int_1^{\infty} e^{-u}du=\frac{1}{2}.$$

Therefore,

$$\sum_{k=1}^{\infty} ke^{-k^2}$$

Converges.

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a.) $$\sum_{n=2}^{\infty}\frac{1}{n^2}=\sum_{n=1}^{\infty}\frac{1}{n^2}-1=\frac{\pi^2}{6}-1.$$

b.) $$\sum_{n=3}^{\infty}\frac{1}{(n+1)^2}=\sum_{n=4}^{\infty}\frac{1}{n^2}=\sum_{n=1}^{\infty}\frac{1}{n^2}-1-\frac{1}{4}-\frac{1}{9}=\frac{\pi^2}{6}-\frac{36-9-4}{36}$$

$$\Rightarrow\sum_{n=3}^{\infty}\frac{1}{(n+1)^2}=\frac{\pi^2}{6}-\frac{49}{36}.$$

c.) $$\sum_{n=1}^{\infty}\frac{1}{(2n)^2}=\frac{1}{4}\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{24}.$$