Homework #2

pg. 197, #30

$$y = \sqrt{\sin(\sqrt{x})}$$
$$= [\sin(x^{1/2})]^{1/2}$$
$$\Rightarrow y' = \frac{1}{2}\sin(x^{1/2})^{-1/2}\cos(x^{1/2}) \cdot \frac{1}{2}x^{-1/2}$$
$$= \frac{1}{4}x^{-1/2}[\sin(x^{1/2})]^{-1/2}\cos(x^{1/2}).$$

pg. 349, #8

a.) $$\int_0^{\pi/2} \frac{d}{dx}\left(\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{3}\right)\right)dx$$
$$= \sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{3}\right)\Big|_0^{\pi/2}$$
$$= \sin\left(\frac{\pi}{4}\right)\cos\left(\pi/6\right)$$
$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2}$$
$$= \frac{\sqrt{6}}{4}.$$

b.) $$\frac{d}{dx}\int_0^{\pi/2} \sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{3}\right)dx = 0.$$

c.) $$\frac{d}{dx}\int_x^{\pi/2} \sin\left(\frac{t}{2}\right)\cos\left(\frac{t}{3}\right)dt = -\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{3}\right).$$

pg. 349, #24

$$\int \frac{x+2}{\sqrt{x^2+4x}}\,dx = \int \frac{x+2}{\sqrt{u}}\,\frac{du}{2(x+2)}$$

$$= \int \frac{1}{2} u^{-1/2}\,du$$

$$= u^{1/2} + C,$$

where $u = x^2+4x$. Consequently,

$$\int \frac{x+2}{\sqrt{x^2+4x}}\,dx = (x^2+4x)^{1/2} + C.$$

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$$\frac{d}{dx}\left[\int_{2x}^{3x+1} \sin(t^4)\,dt\right] = \frac{d}{dx}\left[F(3x+1) - F(2x)\right],$$

where $F'(x) = \sin(x^4)$. Therefore,

$$\frac{d}{dx}\left[\int_{2x}^{3x+1} \sin(t^4)\,dt\right] = 3F'(3x+1) - 2F'(2x)$$

$$= 3\sin((3x+1)^4) - 2\sin(16x^4)$$