# Homework #3

## pg. 363, #50

If the birth rate of a population is

$$b(t) = 2200 + 52.3t + .74t^2$$

people per year and the death rate is

$$d(t) = 1460 + 28.8t$$

people per year find the area between these curves for $0 \le t \le 10$. What does this area represent?

**Solution:**

The area is given by:

$$A = \int_0^{10} [2200 + 52.3t + .74t^2 - 1460 - 28.8t]\,dt$$

$$= \int_0^{10} [740 + 23.5t + .74t^2]\,dt$$

$$= 740t + \frac{23.5t^2}{2} + \frac{.74 \cdot 10^3}{3}$$

$$= 7{,}460 + \frac{2350}{2} + \frac{740}{3}$$

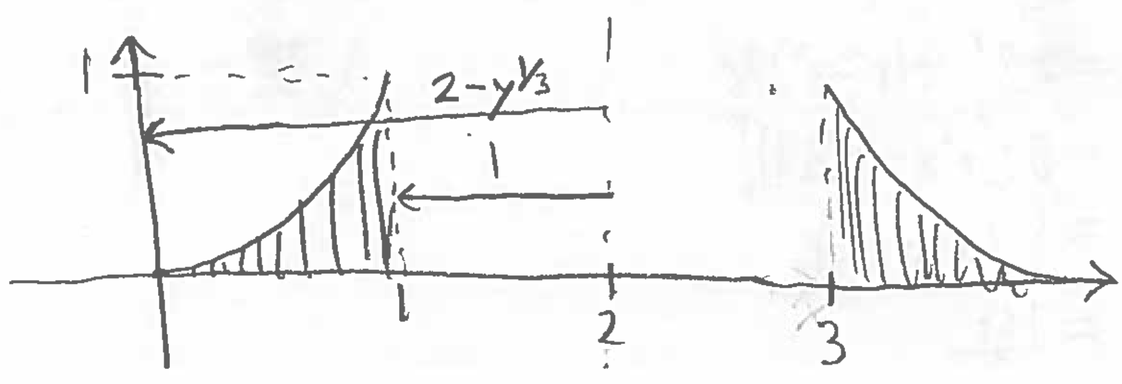
$$= \frac{26{,}465}{3}.$$

This number represents the net change in population.

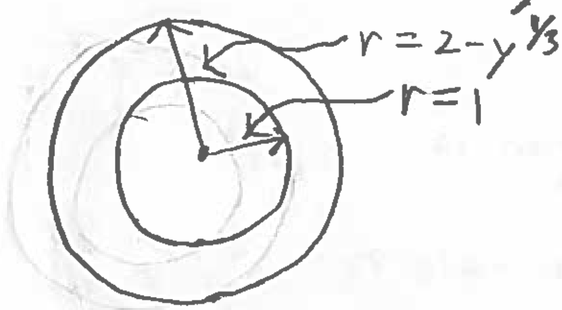
## pg. 376, #16

R = the region bounded by $y = x^3$, $y = 0$, $x = 1$. The solid is formed by revolving R around the line $x = 2$.

**Solution:**



Cross section at coordinate $y$:




The volume is therefore:

$$V = \underbrace{\int_0^1 \pi (2 - y^{1/3})^2\, dy}_{\text{outer circle}} - \underbrace{\pi}_{\text{inner circle}}$$

$$= \int_0^1 [4\pi - 4\pi y^{1/3} + \pi y^{2/3}]\, dy - \pi$$

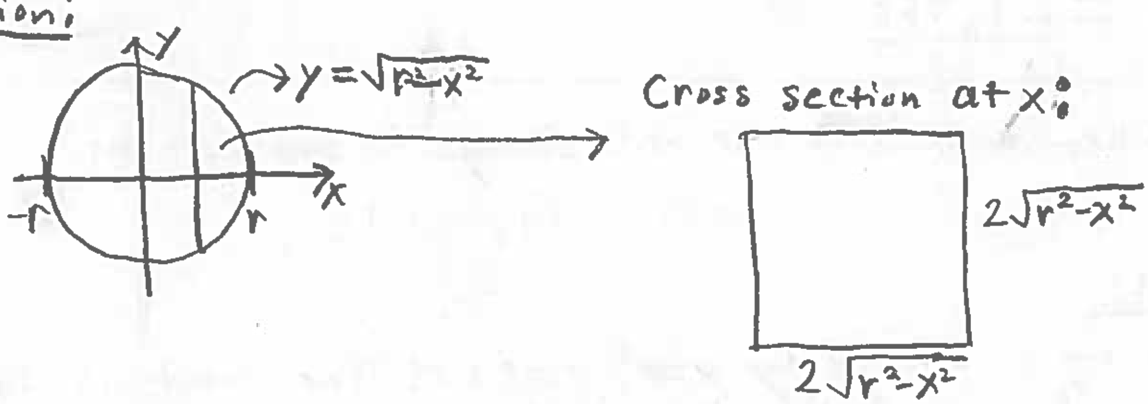
$$= 4\pi - 3\pi + \frac{3}{5}\pi - \pi$$

$$= \frac{3}{5}\pi.$$

Pg 376, #54

The base of $S$ is a circular disk of radius $r$. Parallel cross sections perpindicular to the base are squares.

Solution:




$$V = \int_{-r}^{r} 4(r^2 - x^2)\, dx$$

$$= 2\int_0^r 4(r^2 - x^2)\, dx$$

$$= 8\left[r^2 x - x^3/3\right]\Big|_0^r$$

$$= 8 \cdot \left(r^3 - \frac{r^3}{3}\right)$$

$$= \frac{16r^3}{3}.$$

## pg. 407, #42.

Find $[f^{-1}(3)]'$ if $f(x)=\sqrt{x^3+4x+4}$.

Solution:

$$[f^{-1}(3)]' = \frac{1}{f'(f^{-1}(3))}.$$

However, $f^{-1}(3)=1$. Therefore, since $f'(x)=\frac{1}{2}(x^3+4x+4)^{-1/2}(3x^2+4$ and hence

$$[f^{-1}(3)]' = \frac{1}{f'(1)} = \frac{1}{\frac{1}{2}\cdot\frac{1}{3}\cdot 7} = \frac{6}{7}.$$

$\blacksquare$

## pg. 407, #50

a.) If $f$ is a one-to-one, twice differentiable function with inverse $g$, show that

$$g''(x) = -\frac{f''(g(x))}{[f'(g(x))]^3}$$

b.) Deduce that if $f$ is increasing and concave up, then its inverse is concave down.

$\blacksquare$

Solution:

a.) Since $g(x)=f^{-1}(x)$ it follows that

$$g'(x) = \frac{1}{f'(g(x))}.$$

Therefore,

$$g''(x) = -\frac{f''(g(x))\cdot g'(x)}{[f'(g(x))]^2}$$

$$= -\frac{f''(g(x))\cdot}{[f'(g(x))]^2}\cdot\frac{1}{f'(g(x))}$$

$$= -\frac{f''(g(x))}{[f'(g(x))]^3}.$$

b.) Since $f$ is concave up and increasing it follows that $f'(x) > 0$ and $f''(x) > 0$. Therefore $g''(x) < 0$ and hence concave down.

■