Homework #4

pg. 407, #40

If $f(x)=x^3+3\sin(x)+2\cos(x)$, find $(f^{-1})'(2)$.

Solution:

$f^{-1}(2)=0$ and $f'(x)=3x^2+3\cos(x)-2\sin(x)$. Therefore,

$$(f^{-1})'(2)=\frac{1}{f'(f^{-1}(2))}=\frac{1}{f'(0)}=\frac{1}{3}.$$

$\blacksquare$

pg. 419, #48

If $f(x)=\sqrt{1+xe^{-2x}}$, what is $f'(x)$.

Solution:

$$f'(x)=\frac{1}{2}(1+xe^{-2x})^{-1/2}\cdot(e^{-2x}-2xe^{-2x}).$$

$\blacksquare$

pg. 427, #36

Solve $\ln(2x+1)=2-\ln(x)$.

Solution:

$$\ln(2x+1)=2-\ln(x)$$

$$\Rightarrow 2x+1=e^{2-\ln(x)}$$

$$\Rightarrow 2x+1=\frac{e^2}{x}.$$

$$\Rightarrow 2x^2+x-e^2=0$$

$$x=\frac{-1\pm\sqrt{1+8e^2}}{2}$$

$$x=\frac{-1+\sqrt{1+8e^2}}{2}$$

The negative branch is outside the domain.

$\blacksquare$

pg. 436, #12

If $h(x) = \ln(x+\sqrt{x^2-1})$, calculate $h'(x)$.

Solution:

$$h'(x) = \frac{1}{x+\sqrt{x^2-1}} \cdot \left(1 + \frac{1}{2}(x^2-1)^{-1/2} \cdot 2x\right).$$

pg. 437, #48

If $y = x^{\cos(x)}$, find $y'(x)$.

Solution:

$$\ln(y) = \cos(x)\ln(x)$$

$$\frac{y'}{y} = -\sin(x)\cdot\ln(x) + \frac{\cos(x)}{x}$$

$$y'(x) = \left(-\sin(x)\ln(x) + \frac{\cos(x)}{x}\right)x^{\cos(x)}.$$

pg. 437, #80.

Evaluate $\int \frac{e^x}{e^x+1}\,dx$.

Solution:

Let $u = e^x+1$. Then $du = e^x\,dx$. Consequently,

$$\int \frac{e^x}{e^x+1}\,dx = \int \frac{1}{u}\,du = \ln(|u|) + C = \ln(|e^x+1|) + C.$$