Homework #7

12/10

pg. 531, #10

$$\int_0^{2/3} \sqrt{4-9x^2}\,dx.$$

Solution:

Let $x = \frac{2}{3}\sin\theta$. Then,

$$\int_0^{2/3} \sqrt{4-9x^2}\,dx = 2\int_0^{\pi/2} \cos^2\theta\,d\theta$$

$$= 2\int_0^{\pi/2}\left(\frac{1+\cos(2\theta)}{2}\right)d\theta$$

$$= \theta + \frac{\sin(2\theta)}{2}\Big|_0^{\pi/2}$$

$$= \pi/2 - \frac{1}{2}.$$

pg. 531, #30

$$\int_0^{\pi/2} \frac{\cos(t)}{\sqrt{1+\sin^2 t}}\,dt.$$

Solution:

Let $u = \sin(t)$. Then,

$$\int_0^{\pi/2} \frac{\cos(t)}{\sqrt{1+\sin^2(t)}}\,dt = \int_0^1 \frac{1}{\sqrt{1+u^2}}\,du$$

Let $u = \tan\theta$. Then,

$$\int_0^1 \frac{1}{\sqrt{1+u^2}}\,du = \int_0^{\pi/4} \sec\theta\,d\theta$$

$$= \ln(|\tan\theta + \sec\theta|)\Big|_0^{\pi/4}$$

$$= \ln(|1+\sqrt{2}|) - \ln(|1|)$$

$$= \ln(1+\sqrt{2})$$

pg. 541, #14

$$\int \frac{1}{(x+a)(x+b)}\,dx$$

Solution:

$$\int \frac{1}{(x+a)(x+b)}\,dx = \int\left[\frac{A}{x+a} + \frac{B}{x+b}\right]dx$$

$$= \int \frac{A(x+b)+B(x+a)}{(x+a)(x+b)}\,dx$$

Therefore,

$A = \frac{1}{b-a}$ and $B = \frac{1}{a-b}$

$$\Rightarrow \int \frac{1}{(x+a)(x+b)}\,dx = \frac{1}{b-a}\int\frac{1}{x+a}\,dx + \frac{1}{a-b}\int\frac{1}{x+b}\,dx$$

$$= \frac{1}{b-a}\ln(|x+a|) + \frac{1}{a-b}\ln(|x+b|) + C.$$

pg. 541, #22

$$\int \frac{x^4+9x^2+x+2}{x^2+9}\,dx$$

Solution:

$$
\begin{array}{r l}
 & x^2 + \\
x^2+9 \,\big)\!\! & x^4+0x^3+9x^2+x+2 \\
 & -(x^4+0x^3+9x^2) \\
\hline
 & \text{[illegible]}\ 0 +
\end{array}
$$

$$\Rightarrow \frac{x^4+0x^3+9x^2+x+2}{x^2+9} = x^2 + \frac{x+2}{x^2+9}$$

$$\Rightarrow \int \frac{x^4+9x^2+x+2}{x^2+9}\,dx = \frac{x^3}{3} + \int\frac{x+2}{x^2+9}\,dx$$

$$= \frac{x^3}{3} + \int\frac{x}{x^2+9}\,dx + \int\frac{2}{x^2+9}\,dx$$

$$= \frac{x^3}{3} + \frac{1}{2}\ln(x^2+4) + \frac{2}{9}\int\frac{1}{\left(\frac{x}{3}\right)^2+1}\,dx$$

$$= \frac{x^3}{3} + \frac{1}{2}\ln(x^2+9) + \frac{2}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$$

pg. 547, #16

$$\int_0^{\sqrt{2}/2} \frac{x^2}{\sqrt{1-x^2}}\,dx$$

Solution:

Let $x = \sin\theta$, then

$$\int_0^{\sqrt{2}/2} \frac{x^2}{\sqrt{1-x^2}}\,dx = \int_0^{\pi/4} \sin^2(\theta)\,d\theta$$

$$= \frac{1}{2}\int_0^{\pi/4} \left(1-\cos(2\theta)\right) d\theta$$

$$= \frac{1}{2}\,\pi/4 - \frac{1}{4}\sin(2\theta)\Big|_0^{\pi/4}$$

$$= \frac{\pi}{8} - \frac{1}{4}$$

$\blacksquare$

pg. 547, #22

$$\int \frac{\ln(x)}{x\sqrt{1+[\ln(x)]^2}}\,dx.$$

Solution:

Let $u = \ln(x)$. Then,

$$\int \frac{\ln(x)}{x\sqrt{1+[\ln(x)]^2}}\,dx = \int \frac{u}{\sqrt{1+u^2}}\,du$$

$$= \sqrt{1+u^2} + C$$

$$= \sqrt{1+[\ln(x)]^2} + C.$$

$\blacksquare$