Homework #8

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$$\int_2^\infty ye^{-3y}\,dy = \lim_{R\to\infty}\int_2^R ye^{-3y}\,dy$$

$$= \lim_{R\to\infty}\left(-\tfrac{1}{3}ye^{-y}\Big|_2^R + \int_2^R e^{-y}\,dy\right)$$

$$= 2e^{-2} + \lim_{R\to\infty}\left(-e^{-y}\right)\Big|_2^R$$

$$= 2e^{-2} + 2e^{-2}$$

$$= 3e^{-2}$$

$u = y \qquad v = -\frac{1}{3}e^{-3y}$

$v' = e^{-3y} \qquad u' = 1$

$$-\frac{1}{3}ye^{-3y}\Big|_2^R + \int_2^R \frac{1}{3}e^{-3y}\,dy$$

$$\underbrace{\cancel{-\frac{1}{3}Re^{-3R}}}_{0} + \frac{2}{3}e^{-6} + -\frac{1}{9}e^{-3y}\Big]_2^R$$

$$\frac{2}{3}e^{-6} + \frac{1}{9}e^{-6} = \frac{7}{9}e^{-6}$$

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$$\int_e^\infty \frac{1}{x[\ln(x)]^2}\,dx$$

Let $u = \ln(x) \Rightarrow du = \frac{1}{x}\,dx$

$$\Rightarrow \int_e^\infty \frac{1}{x[\ln(x)]^2}\,dx = \int_1^\infty \frac{1}{u^2}\,du$$

$$= \lim_{R\to\infty} -\frac{1}{u}\Big|_1^\infty$$

$$= 1.$$

7. Let R be the region bounded by the curves $x = 1$, $x = e$, $y = 0$ and the curve $y = \ln(x)$. Find the volume of the solid formed by rotating this region about the y-axis.

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$$\int_0^1 \frac{e^{1/x}}{x^3}\,dx = \lim_{R\to 0^+}\int_R^1 \frac{e^{1/x}}{x^3}\,dx$$

Let $u = 1/x$. Then, $du = -\frac{1}{x^2}dx$

$$\int_0^1 \frac{e^{1/x}}{x^3}\,dx = \lim_{R\to\infty}\int_\infty^1 -ue^u\,du = \lim_{R\to\infty}\int_1^\infty ue^u\,du = \infty.$$

$\blacksquare$

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Decide whether or not the following integral converges:

$$\int_1^\infty \frac{1+\sin^2(x)}{\sqrt{x}}\,dx.$$

Solution:

$$\int_1^\infty \frac{1+\sin^2(x)}{\sqrt{x}}\,dx \geq \int_1^\infty \frac{1}{\sqrt{x}}\,dx = \lim_{R\to\infty}\left(2x^{1/2}\right)\Big|_1^R = \infty.$$

Therefore, $\int_1^\infty \frac{1+\sin^2(x)}{\sqrt{x}}\,dx$ diverges.

$\blacksquare$

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Decide whether or not the following integral converges:

$$\int_0^\infty \frac{\tan^{-1}(x)}{2+e^x}\,dx.$$

Solution:

$$\int_0^\infty \frac{\tan^{-1}(x)}{2+e^x}\,dx \leq \frac{\pi}{2}\int_0^\infty \frac{1}{2+e^x}\,dx \leq \frac{\pi}{2}\int_0^\infty \frac{1}{e^x}\,dx = \frac{\pi}{2}\int_0^\infty e^{-x}\,dx$$

$$\Rightarrow \int_0^\infty \frac{\tan^{-1}(x)}{2+e^x}\,dx \leq \lim_{R\to\infty}\frac{\pi}{2}\left(-e^{-R}+1\right) = \frac{\pi}{2}.$$

Therefore,

$$\int_0^\infty \frac{\tan^{-1}(x)}{2+e^x}\,dx$$

Converges.

$\blacksquare$

6. Evaluate each integral or explain why the integral cannot be done.

- $\displaystyle\int (e^x + 3)\, dx.$

- $\displaystyle\int e^{-x^2}\, dx.$

- $\displaystyle\int \frac{e^x}{e^x + 3}\, dx.$

- $\displaystyle\int \frac{\ln\left(e^{2x}\right)}{x^2}\, dx.$