Math 112
Quiz #10

Compute the following

1. $\sqrt{x}\ln(x)dx$.

$dv=\sqrt{x}$, $u=\ln(x)$

$v=\frac{2}{3}x^{3/2}$, $du=1/x$

$\Rightarrow \int \sqrt{x}\ln(x)dx = \frac{2}{3}x^{3/2}\ln(|x|) - \int \frac{2}{3}x^{1/2}dx$

$= \frac{2}{3}x^{3/2}\ln(|x|) - \frac{4}{9}x^{3/2} + C.$

2. $\int \frac{1}{x^3+x^2+x+1}dx$.

$\int \frac{1}{x^3+x^2+x+1}dx = \int \frac{1}{x^2(x+1)+(x+1)}dx = \int \frac{1}{(x+1)(x^2+1)}dx$

$= \int \left(\frac{A}{x+1} + \frac{Bx+C}{x^2+1}\right)dx$

$\Rightarrow A(x^2+1)+(Bx+C)(x+1)=1$

$\Rightarrow A=1/2$, $B=-1/2$, $C=1/2$

$\Rightarrow \int \frac{1}{x^3+x^2+x+1}dx = \frac{1}{2}\ln(|x+1|) - \frac{1}{4}\ln(|x^2+1|) + \frac{1}{2}\tan^{-1}(x) + C$

3. $\int \frac{x+\sin^{-1}(x)}{\sqrt{1-x^2}}dx$.

$= \int \frac{x}{\sqrt{1-x^2}}dx + \int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}dx$

$= -\frac{1}{2}\sqrt{1-x^2} + \frac{[\sin^{-1}(x)]^2}{2} + C.$