Math 112
Quiz #14

Determine if the following series converges or diverges:

1. $\sum_{k=1}^{\infty} \frac{k^2}{1+k^2}$.

Let $a_k = \frac{k^2}{1+k^2}$. Since $\lim_{k\to\infty} a_k = 1$ it follows that this series diverges

2. $\sum_{k=1}^{\infty} \frac{1}{1+k^2}$

Let $a_k = \frac{1}{1+k^2}$, which is decreasing. Let $f(x) = \frac{1}{1+x^2}$

$$\int_1^{\infty} \frac{1}{1+x^2}dx < \int_1^{\infty} \frac{1}{x^2}dx = 1$$

Therefore,

$$\sum_{k=1}^{\infty} \frac{1}{1+k^2}$$

converges.