Math 112
Quiz #15

For the following two problems assume that $a_n > 0$ and $\sum_{n=1}^{\infty} a_n$ is convergent.

1. Prove that $\sum_{n=1}^{\infty} \ln(a_n)$ is divergent

Since $\sum a_n$ converges it follows that $\lim_{n\to\infty} a_n = 0$.

Therefore,

$$\lim_{n\to\infty} \ln(a_n) = \ln(\lim_{n\to\infty} a_n) = -\infty.$$

Therefore,

$\sum \ln(a_n)$ diverges.

2. Prove that $\sum_{n=1}^{\infty} \tan^{-1}(a_n)$ is convergent.

Let $f(x) = \tan^{-1}(x)$. Then,

$$f'(x) = \frac{1}{1+x^2} < 1 = \frac{d}{dx}x$$

Consequently, since $\tan^{-1}(0) = x(0) = 0$ it follows that

$$\sum_{n=1}^{\infty} \tan^{-1}(a_n) \leq \sum_{n=1}^{\infty} a_n$$

and therefore

$$\sum_{n=1}^{\infty} \tan^{-1}(a_n)$$

converges.