Math 112
Quiz #6

Compute the following:

1. $\int \frac{\sin(x)}{9+\cos^2(x)}dx$

Let $u=\cos(x)$.

$\Rightarrow du=-\sin(x)dx$

$\Rightarrow \int \frac{\sin(x)}{9+\cos^2(x)}dx = -\int \frac{1}{9+u^2}du = -\frac{1}{9}\int \frac{1}{1+(\frac{u}{3})^2}du.$

Let $v=u/3 \Rightarrow dv=du/3$

$\Rightarrow \int \frac{\sin(x)}{9+\cos^2(x)}dx = -\frac{1}{3}\int \frac{1}{1+v^2}dv = -\frac{1}{3}\tan^{-1}\left(\frac{\cos(x)}{3}\right)+C.$

2. $\int e^x \sin(x)dx$

$$I=\int e^x\sin(x)dx = -\cos(x)e^x+\int e^x\cos(x)dx$$
$$= -\cos(x)e^x+e^x\sin(x)-\int e^x\sin(x)dx$$
$$= -\cos(x)e^x+e^x\sin(x)-I$$
$$\Rightarrow 2I = e^x\sin(x)-e^x\cos(x)$$
$$\Rightarrow I = \frac{e^x\sin(x)-e^x\cos(x)}{2}+C.$$