Math 112
Quiz #7

Compute the following:

1. $\int \frac{z^2}{\sqrt{1-z^6}}dz$

Let $u = z^3$. Then, $du = 3z^2 dz$

$$\Rightarrow \int \frac{z^2}{\sqrt{1-z^6}}dz = \frac{1}{3}\int \frac{1}{\sqrt{1-u^2}}du = \frac{1}{3}\sin^{-1}(u) + C = \frac{1}{3}\sin^{-1}(z^3) + C.$$

2. $\int x\sin^2(x^2)dx$

Let $u = x^2$, $du = 2x\,dx$

$$\Rightarrow \int x\sin^2(x^2)dx = \frac{1}{2}\int \sin^2(u)du = \frac{1}{2}\int\left[\frac{1-\cos(2u)}{2}\right]du$$

$$= \frac{1}{2}\left[\frac{x^4}{2} - \frac{\sin(2x^2)}{4}\right] + C$$

3. $\int \sin^5(2t)\cos^2(2t)dt$

$$\frac{1}{2}\int (1-\cos^2(u))^2 \sin(u)\cos^2(u)du, \quad \text{Let } u = 2t$$

$$= -\frac{1}{2}\int [1-v^2]^2 v^2 dv \qquad \text{Let } v = \cos(u)$$

$$= -\frac{1}{2}\int [1-2v^2+v^4]v^2 dv$$

$$= -\frac{1}{2}\int (v^2 - 2v^4 + v^6)dv$$

$$= -\frac{1}{2}\frac{\cos^3(2t)}{3} + \frac{1}{5}\cos^5(2t) - \frac{1}{14}\cos^7(2t) + C.$$