

Math 112

Quiz #9

Compute the following

1. $\int x^3 \sqrt{1-x^2} dx.$

Let $x = \sin \theta$. Then, $dx = \cos \theta d\theta$

$\Rightarrow \int x^3 \sqrt{1-x^2} dx = \int \sin^3 \theta \cos^2 \theta d\theta$

$= \int (1 - \cos^2 \theta) \sin \theta \cos^2 \theta d\theta$

$u = \cos \theta$

$du = -\sin \theta d\theta$

$\Rightarrow \int x^3 \sqrt{1-x^2} dx = \int (u^2 - 1) u^2 du = \int u^4 - u^2 du = \frac{(\sqrt{1-x^2})^5}{5} - \frac{(\sqrt{1-x^2})^3}{3} + C$

2. $\int \frac{3x}{x^2 - 3x - 4} dx.$

$\int \frac{3x}{x^2 - 3x - 4} dx = \int \frac{A}{x-4} dx + \int \frac{B}{x+1} dx = \frac{12}{5} \int \frac{1}{x-4} dx + \frac{3}{5} \int \frac{1}{x+1} dx$

$\Rightarrow A(x+1) + B(x-4) = 3x$ $= \frac{12}{5} \ln(|x-4|) + \frac{3}{5} \ln(|x+1|) + C$

$x = -1 \Rightarrow B = \frac{3}{5}$

$x = 4 \Rightarrow 5A = 12$

$A = \frac{12}{5}$