MST 352/652
Homework #4

Due Date: February 12, 2019

1 Problems for everyone

1. pg. 72, #3.1.2, #3.1.4-3.1.6.
2. Consider the following initial boundary value problem on the domain $[0, \pi]$:

$$\begin{cases} tu_t = u_{xx} + 2u \\ u(t,0) = u(t,\pi) \\ u(0,x) = 0 \end{cases}.$$

   By separating variables, show that this initial boundary value problem has an infinite number of solutions.

3. Let $l, k > 0$. Find all separable eigensolutions of the Schrödinger equation $u_t = iku_{xx}$ on the interval $[0, l]$ subject to following boundary conditions:

$$u_x(t,0) = 0 \text{ and } u(t,l) = 0.$$

4. The overdamped dynamics of an elastic beam on the interval $[0, l]$, i.e. a thin plank of wood of length l, can be modeled by the following initial boundary value problem:

$$\begin{cases} u_t = u_{xxxx} \\ u(0,x) = f(x) \\ u(t,0) = u(t,l) = 0 \\ u_{xx}(t,0) = u_{xx}(t,l) = 0 \end{cases}.$$

   (a) Find all separable eigensolutions to this equation.
   (b) What does the initial condition model?
   (c) What do the boundary conditions model?

5. pg. 76, #3.2.1-3.2.4.
6. Let f be a periodic function of period p.
   (a) Prove that for any $a \in \mathbb{R}$:

$$\int_0^p f(x)\,dx = \int_a^{a+p} f(x)\,dx.$$

   Hint: Write $\int_a^{a+p} f(x)\,dx$ as the sum of two integrals (a to p, and p to $a+p$) and make an appropriate change of variables.

(b) Prove that for any $a \in \mathbb{R}$:

$$\int_0^p f(x+a)\,dx = \int_0^p f(x)\,dx.$$

(c) Interpret these identities graphically.

2 Graduate Problems

1. #3.1.7

2. Let $l > 0$. Consider the wave equation on the domain $[0, l]$ with so called Robin boundary conditions:

$$\begin{cases} u_{tt} = u_{xx} \\ u(0,x) = f(x) \\ u_t(0,x) = g(x) \\ u_x(t,0) = u(t,0) \\ u_x(t,l) = u(t,l) \end{cases}.$$

   By constructing an appropriate energy, prove that solutions to this initial boundary value problem are unique.

   Hint: The energy I defined in class won't work, but it might provide hints on how to add additional terms that yield an energy that is conserved for all time.

3. #3.2.10