MST 352/652
Homework #8

Due Date: March 19, 2019

1 Problems for everyone

1. pg. 149-151, #4.2.3, #4.2.4, #4.2.7, #4.2.8
2. Consider the following initial-boundary value problem:

$$\begin{aligned} &u_t = u_{xx},\ x \in [0, 2\pi], \\ &u(0,t) = 1,\ u(2\pi,t), = 2 \\ &u(x,0) = x. \end{aligned}$$

   (a) Calculate the steady state solution for this initial-boundary value problem.

   (b) Solve this initial boundary value problem.

3. Consider the following initial-boundary value problem:

$$\begin{aligned} &u_t = u_{xx} - u,\ x \in [0, 1], \\ &u(0,t) = u(1,t) = 0, \\ &u(x,0) = x(1-x). \end{aligned}$$

   (a) Using separation of variables solve this initial-boundary value problem.

   (b) Using your solution, calculate $\lim_{t\to\infty} u(x,t)$.

   (c) Are there any steady state solutions to this equation? If so, what are they?

4. Consider the following initial-boundary value problem:

$$\begin{aligned} &u_{tt} = u_{xx},\ x \in [0, \pi], \\ &u_x(0,t) = u_x(\pi,t) = 0, \\ &u(x,0) = \cos^2(x), \\ &u_t(x,0) = \cos(3x). \end{aligned}$$

   (a) Solve this initial-boundary value problem.

   (b) Using software such as Matlab, Mathematica, etc sketch the solution for $t = 0$, $t = \pi/2$, $t = \pi$, $t = 3\pi/2$, and $t = 2\pi$.

   (c) Sketch a contour plot of your solution as a function of x and t. (If you want to, you can use software to do this.)

   (d) Describe qualitatively the behavior of the solution.

2 Graduate Problems

1. pg. 149-151, #4.2.10, #4.2.11
2. Consider the following initial-boundary value problem:

$$\begin{aligned} &u_{tt} + u_t = u_{xx},\ x \in [0, \pi], \\ &u(0, t) = u(\pi, t) = 0, \\ &u(x, 0) = \sin^2(x), \\ &u_t(x, 0) = 0. \end{aligned}$$

   (a) Solve this initial-boundary value problem.

   (b) Using software such as Matlab, Mathematica, etc sketch the solution for $t = 0$, $t = \pi/2$, $t = \pi$, $t = 3\pi/2$, and $t = 2\pi$ using 20 terms in the Fourier series.

   (c) Sketch a contour plot of your solution as a function of x and t. (If you want to, you can use software to do this.)

   (d) Describe qualitatively the behavior of the solution.