Partial Differential Equations, Spring 2019
MST-352/652

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Course Website: http://users.wfu.edu/gemmerj/math352-652.html
Office Hours: Monday 1-3, Tuesday 2-3, Wednesday 3-5
Class Meeting Times: TR 9:30-10:45
Class Location: Manchester Hall 124

Textbook: Olver, Peter J. *Introduction to partial differential equations*. Switzerland: Springer, 2014.

Prerequisites: Multivariable calculus (MST 113) and ordinary differential equations (MST 251) are required. Linear algebra (MST 121) is also strongly encouraged. In addition, some familiarity with with computer programming of numerical algorithms (MATLAB or Mathematica code usually) is useful but not required.

Course Description: An introduction to partial differential equations with emphasis on developing classic solution techniques for the heat equation, wave equation and Laplace's equation on unbounded and bounded domains in one or two spatial dimensions. We will also spend some time on methods for numerically solving partial differential equations in Matlab. Specific topics covered will include: method of characteristics, Fourier series, eigenfunction expansions, finite difference approximations, Fourier and Laplace transforms, Duhamel's principle, eigenfunction expansions, Green's functions, method of characteristics, and traveling wave solutions.

Course Rationale: Many physical processes involve quantities that vary in *space and time*. For example, the temperature in a room being heating by a fire varies not only in time but with distance from the heat source (heat equation) and the amplitude of a sound wave fluctuates periodically both in time and space (wave equation). Many other physical processes vary in more than one spatial dimension. For example, the equilibrium potential of an electrostatic field in a domain free from charges (Laplace's equation). Mathematical models of such phenomenon consist of differential equations with partial derivatives, i.e. partial differential equations. This course will provide an introduction to the basic properties of partial differential equations and to some mathematical techniques useful in analyzing them. Along the way, we will discuss many applications including diffusion, propagation of waves, electrostatics, conservation laws, and reaction diffusion equations. While I will motivate all concepts by their underlying physics the focus will be on the mathematics.

Course Delivery: The course material will be delivered through a combination of lectures and in class group assignments. Evaluation of the students understanding of the material will be assessed through written homework assignments, in-class exams, computational assignments and for graduate students a term paper.

Course Policies:

✦ **Grading:** Your grade will be based on:

- Weekly written homework: 20%
- Two in-class exams: 30% (15% each)
- Final Exam: 30%
- **Graduation Students:** Term Paper and Computational Assignments 20%
- **Undergraduate Students:** Computational Assignments and Optional Term Paper: 20-25%

You are guaranteed the following grades if your final percentage lies within the following ranges:

90-92.9: A-	**93-100: A**	
80-82.9: B-	**83-86.9: B**	**87-89.9: B+**
70-72.9: C-	**73-76.9: C**	**77-79.9: C+**
60-62.9: D-	**63-66.9: D**	**67-69.9: D+**

✦ **Written homework:** Written homework will be assigned most weeks on Tuesday and will be due Tuesday at the beginning of class the following week. The assigned homework problems will be posted on the course website. Late homework will not be accepted under any circumstances. However, I will drop the lowest homework score from your grade. Written homework must consist of solutions that show all steps, be your own work and be written clearly using complete sentences as appropriate (see homework policy).

✦ **In-Class Exams:** There will be two in-class exams and a comprehensive final in the course.

✦ **Computational Assignments:** Throughout the course there will be two computational assignments. These assignments will consist of in class group work in which students will learn how to numerically solve various partial differential equations in Matlab. The students will then complete an out of class component which will be due within at least a week. The numerical assignments will be submitted on Sakai and must consist of Matlab script files.

✦ **Term Paper:** Every graduate student is required to write a term paper on a topic in partial differential equations from the textbook that we will not cover in the course. The paper must be written in LaTeX. Your textbook is written for a two semester course in partial differential equations and the later chapters have a wealth of interesting topics to pick from. In addition to writing an expository paper, you will be required to work through some of the problems from the text. Undergraduate students can write a term paper for a possible 5% bonus.

Possible Topics:

1. Finite element methods.
2. Partial differential equations in two or more spatial dimensions.
3. General framework for linear PDEs
4. Nonlinear evolution equations.

Tentative Course Schedule:

1. 1/15: What are partial differential equations, **Chapter 1**.
2. 1/17: Linear operators, **Chapter 1.**
3. 1/22: Linear transport equation, **Sections 2.1-2.2**
4. 1/24: The wave equation and D'Alembert's formula, **Section 2.4**
5. 1/29: Eigensolutions of linear equations, **Section 3.1**
6. 1/31: Fourier series Part 1, **Section 3.2**
7. 2/05: Fourier series Part 2, **Section 3.2**
8. 2/07: Calculus of Fourier series, **Sections 3.3-3.4**
9. 2/12: Convergence of Fourier series, **Section 3.5**
10. 2/14: **Exam #1**

11. 2/19: Heat equation Part 1, **Section 4.1**
12. 2/21: Heat equation Part 2, **Section 4.1**
13. 2/26: Finite difference approximations, **Section 5.1**
14. 2/28: Computational assignment on the heat equation, **Section 5.2**
15. 3/05: Wave equation Part 1, **Section 4.2**
16. 3/07: Computational assignment on the wave equation, **Section 5.3**
17. 3/19: Laplace's equation, Part 1, **Section 4.3**
18. 3/21: Laplace's equation, Part 2, **Section 4.3**
19. 3/26: Classification of PDEs, **Section 4.4**
20. 3/28: **Exam #2**

21. 4/02: Generalized functions, **Section 6.1**
22. 4/04: Green's functions Part 1, **Section 6.2**
23. 4/09: Green's functions Part 2, **Section 6.2**
24. 4/11: Fourier Transform, **Section 7.1**
25. 4/16: Calculus of the Fourier Transform, **Section 7.2**
26. 4/18: Green's functions and convolutions, **Section 7.3**
27. 4/23: Fundamental solution to the heat equation, **Section 8.1**
28. 4/25: The maximum principle, **Section 8.3**
29. 4/30: Bonus day

Important Dates:

1. February 14: Exam 1.
2. February 19: Last day to drop.
3. March 9-17: Spring break.
4. March 28: Exam 2.
5. May 6: Final Exam at 2:00

The Honor Code: At Wake Forest, we expect you to behave as honorable citizens of the class, the university, and the world as a whole. When you complete an assignment with your name on it, you are representing that everything you are turning in is your own work. That means that you do not copy from other students, textbooks, or websites. If at any time I become aware of cheating or plagiarism in this course, I will submit the information to the honor council.