

MST 352/652

Term Paper: Planar Waves

Due Date: May 06, 2019

The goal of this project is to understand the dynamics of solutions to the heat and wave equation in rectangular and circular domains. As in one spatial dimension, separation of variables can be applied with the resulting eigenfunctions consisting of trigonometric functions (rectangular domains) or Bessel functions (circular domains). The analysis of the dynamics on rectangular domains is a direct extension of what we have done in one spatial dimension. However, on circular domains the analysis of Bessel functions is not as straightforward. Bessel functions are only orthogonal with respect to a weighted inner product. The goal of your term paper is to learn how to use Bessel functions to solve linear PDEs on circular domains. Since the calculations can be quite complicated, the focus should be on the big picture in particular the generic shape of the eigenfunctions.

Your term paper should give a recap of the section(s) you read from the text as well as the solutions to the required problems. Some of the key concepts are given below as well as the relevant sections from the text and homework problems.

1. **Textbook Sections:** pg. 445: Explicit solutions of the heat equation, pg. 466: Bessel's equation, pg. 474: Heat Equation in a Disk, pg. 486: The Planar Wave Equation.
2. **Key Concepts:** Separation of variables, Helmholtz equation, eigensolutions, Bessel functions, Bessel equation, Bessel roots, normal modes, Fourier Bessel series, Biorgot's theorem, nodal curves.
3. **Problems:** #11.2.1, #11.2.6, #11.2.12, #11.4.4, #11.4.10, #11.6.21, #11.6.38, #11.6.41, #11.6.42, #11.6.43, #11.6.45.