MST 352/652
Term Paper: Nonlinear Shock Dynamics

Due Date: May 06, 2019

The goal of this project is to understand the dynamics of solutions to the following PDE:

$$u_t + c(u)u_x = 0. \tag{1}$$

Formally, this equation can be solved using the method of characteristics. However, since the wave speed depends on the state of the system the solution can become multivalued. The resolution of this apparent contradiction is to realize that (1) results from an integral conservation law. That is, (1) is the differential form of an integral equation and thus assumes more regularity, i.e. differentiability, than the actual problem requires. In this term paper you will learn how to patch together local solutions of (1) in a manner that satisfies the underlying integral equation. The resulting non-smooth solutions will consist of *shock* and *rarefaction* waves.

Your term paper should give a recap of the section(s) your read from the text as well as the solutions to the required problems. Some of the key concepts are given below as well as the relevant sections from the text and homework problems.

1. **Textbook Sections:** pg. 31: Nonlinear Transport and Shocks.
2. **Key Concepts:** Method of characteristics, conservation laws, rarefaction waves, shock waves, Rainkine-Hugonit codition, entropy condition.
3. **Problems:** pg. 47-49, #2.3.2-2.3.4, #2.3.15-2.3.18.