

Homeworks #1.

pg. 15, #2.

$$\begin{array}{l} 2x + y - 2z = 0 \\ 2x - y - 2z = 0 \\ x + 2y - 4z = 0 \end{array} \quad \left[\begin{array}{ccc|c} 2 & 1 & -2 & 0 \\ 2 & -1 & -2 & 0 \\ 1 & 2 & -4 & 0 \end{array} \right] \begin{array}{l} \uparrow \\ \\ \downarrow \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -4 & 0 \\ 2 & -1 & -2 & 0 \\ 2 & 1 & -2 & 0 \end{array} \right] \begin{array}{l} \\ -2R1 \\ -2R1 \end{array} \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -4 & 0 \\ 0 & -5 & 6 & 0 \\ 0 & -3 & 6 & 0 \end{array} \right] \begin{array}{l} \\ \\ \uparrow -3 \downarrow \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -4 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & -5 & 6 & 0 \end{array} \right] \begin{array}{l} \\ \\ +5R2 \end{array} \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -4 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & -4 & 0 \end{array} \right]$$

$$\Rightarrow x = y = z = 0.$$

pg. 15, #6.

$$\begin{array}{l} 2x + 3y + z = 4 \\ x + 9y - 4z = 2 \\ x - y + 2z = 3 \end{array} \quad \left[\begin{array}{ccc|c} 2 & 3 & 1 & 4 \\ 1 & 9 & -4 & 2 \\ 1 & -1 & 2 & 3 \end{array} \right] \begin{array}{l} \uparrow \\ \\ \downarrow \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 1 & 9 & -4 & 2 \\ 2 & 3 & 1 & 4 \end{array} \right] \begin{array}{l} \\ -R1 \\ -2R1 \end{array} \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 0 & 10 & -6 & -1 \\ 0 & 5 & -3 & -2 \end{array} \right] \begin{array}{l} \\ \\ -2R3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 0 & 0 & 0 & 3 \\ 0 & 5 & -3 & -2 \end{array} \right] \quad 3 \neq 0 \Rightarrow \text{No solution.}$$

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$$\left[\begin{array}{cccc|c} 1 & -1 & 1 & 1 & a \\ 1 & 1 & -2 & 3 & b \\ 3 & -2 & 3 & -2 & c \\ 0 & 2 & -3 & 2 & d \end{array} \right] \begin{array}{l} -R_1 \\ -3R_1 \end{array} \Rightarrow \left[\begin{array}{cccc|c} 1 & -1 & 1 & 1 & a \\ 0 & 2 & -3 & 2 & b-a \\ 0 & 1 & 0 & -5 & c-3a \\ 0 & 2 & -3 & 2 & d \end{array} \right] \begin{array}{l} \uparrow \\ \downarrow \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 1 & 1 & a \\ 0 & 1 & 0 & -5 & c-3a \\ 0 & 2 & -3 & 2 & b-a \\ 0 & 2 & -3 & 2 & d \end{array} \right] \begin{array}{l} -2R_2 \\ -2R_2 \end{array} \quad \left[\begin{array}{cccc|c} 1 & -1 & 1 & 1 & a \\ 0 & 1 & 0 & -5 & c-3a \\ 0 & 0 & -3 & 12 & b-a+2c+6a \\ 0 & 0 & -3 & 12 & d-2c+6a \end{array} \right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & -1 & 1 & 1 & a \\ 0 & 1 & 0 & -5 & c-3a \\ 0 & 0 & -3 & 12 & b-a-2c+6a \\ 0 & 0 & 0 & 0 & d-2c+6a-b+a+2c-6a \end{array} \right]$$

To be consistent we need
 $d = b - a$.

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$$\begin{array}{l} x+y+2z=0 \\ 3x-y-2z=0 \\ 2x-2y-4z=0 \\ x+3y+6z=0 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 3 & -1 & -2 & 0 \\ 2 & -2 & -4 & 0 \\ 1 & 3 & 6 & 0 \end{array} \right] \begin{array}{l} -3R_1 \\ -2R_1 \\ -R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & -4 & -8 & 0 \\ 0 & -4 & -8 & 0 \\ 0 & 2 & 4 & 0 \end{array} \right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

\Rightarrow Nontrivial Solutions.

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- $x + y = 1$ has one
 $2x + 2y = 2$ solution.
 $x + 2y = 1$

- $x + y = 1$ has infinitely
 $2x + 2y = 2$ many solutions: $x = -y$
 $3x + 3y = 3$

- $x + y = 1$ has no
 $2x + 2y = 1$ solutions.
 $3x + 3y = 1$