Homework #5

~~pg.~~ 93, #6

$$c_1\begin{bmatrix}0\\4\\-1\end{bmatrix}+c_2\begin{bmatrix}1\\5\\-3\end{bmatrix}+c_3\begin{bmatrix}1\\-3\\-1\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\Rightarrow\left[\begin{array}{ccc:c}0&1&1&0\\4&5&-3&0\\-1&-3&-1&0\end{array}\right]+4R3 \Rightarrow \left[\begin{array}{ccc:c}-1&-3&-1&0\\0&-7&-7&0\\0&1&1&0\end{array}\right]+7R2 \rightarrow \left[\begin{array}{ccc:c}-1&-3&-1&0\\0&0&0&0\\0&1&1&0\end{array}\right]$$

$\Rightarrow c_1=c_2=c_3=0 \Rightarrow$ linearly dependent.

~~pg.~~ 93, #10

$$c_1(x^3-1)+c_2(x^2-1)+c_3(x-1)+c_4=0$$

$$\Rightarrow\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\-1&-1&-1&1\end{bmatrix}\begin{bmatrix}c_1\\c_2\\c_3\\c_4\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\-1&-1&-1&1\end{bmatrix}+R1+R2+R3 \rightarrow \begin{bmatrix}1&&&\\&1&&0\\&0&1&\\&&&1\end{bmatrix}$$

$\Rightarrow c_1=c_2=c_3=c_4=0$

$\Rightarrow$ linearly ind.

~~pg.~~ 93, #18

$$c_1(x^3+x)+c_2(x^2-x)+c_3(x+1)+c_4(x^3+1)=0$$

$$\Rightarrow\left[\begin{array}{cccc:c}1&0&0&1&0\\0&1&0&0&0\\1&-1&1&0&0\\0&0&1&1&0\end{array}\right]-R3 \Rightarrow \left[\begin{array}{cccc:c}1&0&0&1&0\\0&1&0&0&0\\0&-1&1&-1&0\\0&0&1&1&0\end{array}\right]+R3$$

$$\left[\begin{array}{cccc:c}1&0&0&1&0\\0&1&0&0&0\\0&0&1&-1&0\\0&0&1&1&0\end{array}\right]-R3 \Rightarrow \left[\begin{array}{cccc:c}1&0&0&1&0\\0&1&0&0&0\\0&0&1&-1&0\\0&0&0&2&0\end{array}\right]$$

$\rightarrow$ The vectors are linearly ind. Since $\dim(P_3)=4$ it follows these polynomials are a basis.

pg. 94, #24

$$V=\begin{bmatrix}-1\\1\\0\end{bmatrix}=c_1\begin{bmatrix}2\\-1\\0\end{bmatrix}+c_2\begin{bmatrix}1\\3\\-1\end{bmatrix}+c_3\begin{bmatrix}1\\-4\\-1\end{bmatrix}$$

$$\rightarrow\left[\begin{array}{ccc|c}2&1&1&-1\\-1&3&-4&1\\0&-1&-1&0\end{array}\right]+2R2\rightarrow\left[\begin{array}{ccc|c}0&7&-7&1\\-1&3&-4&1\\0&-1&-1&0\end{array}\right]+7R3$$

$$\rightarrow\left[\begin{array}{ccc|c}0&0&-14&1\\-1&3&-4&1\\0&-1&-1&0\end{array}\right]$$

$c_3=-1/14$

$c_2=1/14$

$-c_1+\frac{3}{14}+\frac{4}{14}=0\Rightarrow c_1=1/2$

Therefore,

$$[V]_p=\begin{bmatrix}1/2\\1/14\\-1/14\end{bmatrix}$$

pg. 105, #14

If $\begin{bmatrix}a\\b\\c\end{bmatrix}$ is in span of $\{\vec{v}_1,\vec{v}_2,\vec{v}_3,\vec{v}_4\}$ it follows that:

$$\begin{bmatrix}a\\b\\c\end{bmatrix}=c_1\vec{v}_1+c_2\vec{v}_2+c_3\vec{v}_3+c_4\vec{v}_4$$

$$\Rightarrow\left[\begin{array}{cccc|c}0&1&2&4&a\\-2&-1&0&-2&b\\1&0&-1&-1&c\end{array}\right]+2R3\rightarrow\left[\begin{array}{cccc|c}0&1&2&4&a\\0&-1&-2&-4&b+2c\\1&0&-1&-1&c\end{array}\right]+R2$$

$$\Rightarrow\left[\begin{array}{cccc|c}0&0&0&0&a+b+2c\\0&-1&-2&-4&b+2c\\1&0&-1&-1&c\end{array}\right]\Rightarrow a=-b-2c$$

$$\Rightarrow\begin{bmatrix}a\\b\\c\end{bmatrix}=\begin{bmatrix}-b-2c\\b\\c\end{bmatrix}=b\begin{bmatrix}-1\\1\\0\end{bmatrix}+c\begin{bmatrix}-2\\0\\1\end{bmatrix}$$

$$\Rightarrow\left\{\begin{bmatrix}-1\\1\\0\end{bmatrix},\begin{bmatrix}-2\\0\\1\end{bmatrix}\right\}$$

is a basis.