Homework #8

#8.

$xy'+y=x^2,\quad x<0$

$\Rightarrow \frac{d}{dx}(xy)=x^2$

$\Rightarrow xy = x^3/3 + c$

$$\boxed{y=\frac{x^2}{3}+\frac{c}{x}}$$

#14

$2xy'+y=1$

$y(4)=0$

$\Rightarrow y'+\frac{1}{2x}y=\frac{1}{2x}$

$e^{f(x)}y'+\frac{e^{f(x)}}{2x}y=\frac{e^{f(x)}}{2x}$

$\frac{d}{dx}(e^{f(x)}y)=y'e^{f(x)}+f'(x)e^{f(x)}y=e^{f(x)}y'+\frac{e^{f(x)}}{2x}y$

$\Rightarrow f'(x)=\frac{1}{2x}\Rightarrow f(x)=\frac{1}{2}\ln(x)=\ln(x^{1/2})$

$\Rightarrow e^{f(x)}=x^{1/2}$

Therefore,

$\frac{d}{dx}(x^{1/2}y)=\frac{x^{1/2}}{2x}=\frac{1}{2x^{1/2}}$

$\Rightarrow x^{1/2}y=x^{1/2}+c$

$\Rightarrow y=1+\frac{c}{x^{1/2}}$

$y(4)=1+\frac{c}{2}=0$

$\Rightarrow c=-2$

$$\boxed{y=1-\frac{2}{x^{1/2}}}$$

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$$x^2y'' + xy' - y = x$$
$$y(1) = 0$$
$$y'(1) = -1$$

$$y_p(x) = -\frac{x}{4} + \frac{1}{2}x\ln(x)$$

$$y_p'(x) = -\frac{1}{4} + \frac{1}{2}\ln(x) + \frac{1}{2} = \frac{1}{4} + \frac{1}{2}\ln(x)$$

$$y_p''(x) = \frac{1}{2x}$$

Therefore,

$$x^2y_p'' + xy_p' - y_p = \frac{x}{2} + \frac{x}{4} + \frac{x}{2}\ln(x) + \frac{x}{4} - \frac{1}{2}x\ln(x)$$
$$= x.$$

The general solution is therefore

$$y = \frac{c_1}{x} + c_2x - \frac{x}{4} + \frac{1}{2}x\ln(x)$$

$$y(1) = c_1 + c_2 - \frac{1}{4} = 0$$

$$y'(x) = -\frac{c_1}{x^2} + c_2 - \frac{1}{4} + \frac{1}{2}\ln(x) + \frac{1}{2}$$

$$y'(1) = -c_1 + c_2 + \frac{1}{4} = -1$$

$$\Rightarrow c_1 + c_2 = 1/4$$
$$-c_1 + c_2 = -5/4$$

Solve for c_1, c_2, ...

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#4.

$2y'' - 5y' - y = 0$

$y = e^{\lambda x}$

$\Rightarrow 2\lambda^2 - 5\lambda - 1 = 0$

$\Rightarrow \lambda = \dfrac{5 \pm \sqrt{25+8}}{4}$

$= \dfrac{5 \pm \sqrt{33}}{4}$

$y = c_1 \exp\left(\dfrac{5+\sqrt{33}}{4} x\right) + c_2 \exp\left(\dfrac{5-\sqrt{33}\,x}{4}\right)$

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#12.

$2y'' - 8y' + 14y = 0$

$y = e^{\lambda x}$

$2\lambda^2 - 8\lambda + 14 = 0$

$\lambda^2 - 4\lambda + 7 = 0$

$\lambda = \dfrac{4 \pm \sqrt{16-28}}{2}$

$= \dfrac{4 \pm \sqrt{-12}}{2}$

$= 2 \pm \sqrt{3}\, i$

$y = c_1 e^{2x} e^{\sqrt{3} i x} + c_2 e^{2x} e^{-\sqrt{3} i x}$

or

$y = c_1 e^{2x} \cos(\sqrt{3}x) + c_2 e^{2x} \sin(\sqrt{3}x).$