

Math 205
Quiz #6

1. Consider the following differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 8y = 0.$$

(a) Find the general solution to this problem.

$$\begin{aligned}y &= e^{\lambda x} \\ \Rightarrow \lambda^2 e^{\lambda x} + 4\lambda e^{\lambda x} + 8e^{\lambda x} &= 0 \\ \Rightarrow \lambda^2 + 4\lambda + 8 &= 0 \\ \Rightarrow \lambda &= \frac{-4 \pm \sqrt{16-32}}{2} \\ &= -2 \pm 2i \\ y_H &= c_1 e^{-2x} \cos(2x) + c_2 e^{-2x} \sin(2x)\end{aligned}$$

(b) Find the solution to this problem if the initial conditions are given by

$$y(0) = 0 \text{ and } y'(0) = -1.$$

$$\begin{aligned}y(0) &= c_1 = 0 \\ y'(x) &= -2c_2 e^{-2x} \sin(2x) + 2c_2 e^{-2x} \cos(2x) \\ y'(0) &= 2c_2 = -1 \\ \Rightarrow c_2 &= -\frac{1}{2} \\ y(x) &= -\frac{1}{2} e^{-2x} \sin(2x).\end{aligned}$$