MST 750
Homework #7

Due Date: March 25, 2022

1. Can $\phi(t) = (\sin(t), \sin(2t))$ be the solution of an autonmous system $\dot{x} = f(x)$?

2. Let $f : \mathbb{R}^n \mapsto \mathbb{R}^n$ be a function of class C^∞ such that the equation $\dot{x} = f(x)$ defines a flow φ_t in $\mathbb{R}^n$.

   (a) Show that
   $$\varphi_t(x) = x + f(x)t + \frac{1}{2}\nabla f(x) f(x) t^2 + o(t^2).$$

   (b) Verify that
   $$\det(\nabla \varphi_t(x)) = 1 + \nabla \cdot f(x)t + o(t).$$

   (c) Given an open set $A \subset \mathbb{R}^n$ and $t \in \mathbb{R}$ show that
   $$\frac{d}{dt}\mu(\varphi_t(A)) = \int_{\varphi_t(A)} \nabla \cdot f(x) dV,$$
   where μ denotes the volume of a set in $\mathbb{R}^n$. **Hint:** First argue that
   $$\frac{d}{dt}\mu(\varphi_t(A)) = \int_{\partial\varphi_t(A)} f(x) \cdot \ltimes dA,$$
   where $\partial\varphi_t(A)$ denotes the boundary of $\varphi_t(A)$ with outward normal $\mathbf{n}$. You do not have to do this argument rigorously and a version of this argument can be found in Strogatz.

   (d) Show that if $\nabla \cdot f(x) = 0$, then the equation $\dot{x} = f(x)$ has neither asymptotically stable fixed points nor asymptotically stable periodic solutions.

3. pg. 156, #12

4. pg. 156, #13

5. pg. 156, #14

6. pg. 157, #17

7. pg. 157, #18