Dynamical Systems

MST 750

Dr. John Gemmer: gemmerj@wfu.edu
Office: Manchester 388
Course Website: http://users.wfu.edu/gemmerj/math750S22.html
Canvas: The course syllabus and grades will be posted on Canvas
Office Hours: T 2:00-3:00, W 2:00-4:00, Th 3:00-5:00
Class Meeting Times: MWF 11:00-11:50
Class Location: Carswell 101

COURSE DESCRIPTION

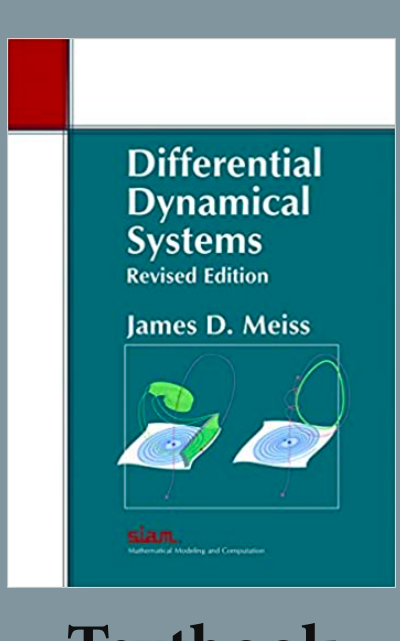
Classically, dynamical systems arose out of the need to describe the dynamics of physical systems such as planetary orbits and electronic circuits but now is a critical tool to understanding the dynamics of climate, epidemics, etc. This course serves as introduction to the modern theory of dynamical systems. It will combine tools from analysis, differential geometry, and applied mathematics to study the qualitative behavior of solutions to differential equations. Specific topics covered will include linear and nonlinear autonomous differential equations, invariant sets, closed orbits, Poincare maps, structural stability, center manifolds, normal forms, local bifurcations of equilibria, linear and non-linear maps, hyperbolic sets, attractors, Hamiltonian dynamics, and KAM theory.

COURSE RATIONALE

This course will serve the needs of both pure and applied mathematicians. Nominally the course could be classified as applied analysis but it will also synthesize and apply techniques from disparate areas of mathematics: differential equations, linear algebra, analysis, topology, and numerical methods to be precise. This course will not just be an abstract treatment of ODEs but the theory will be placed within the context of its many applications to physics, biology, chemistry, and engineering. Furthermore, to develop a geometrical understanding of dynamics, each student must experiment. Simple codes will be written in Mathematica and Matlab which will allow the students to explore various dynamical systems.

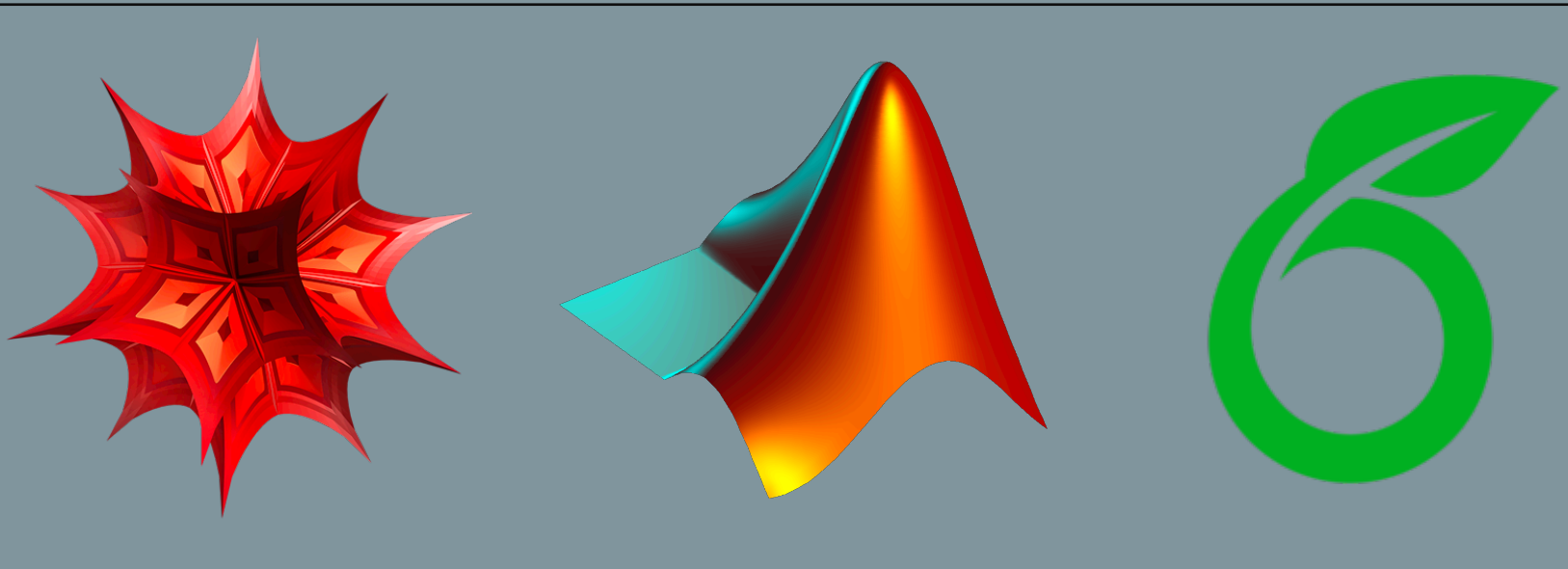
REQUIREMENTS

Prerequisites: Analysis at the MST 311/611 level



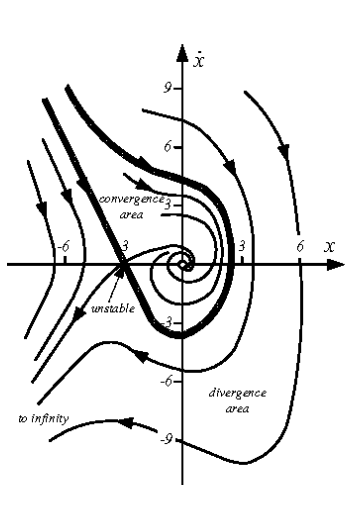
Textbook James Meiss

Computer Access: Laptop/Desktop

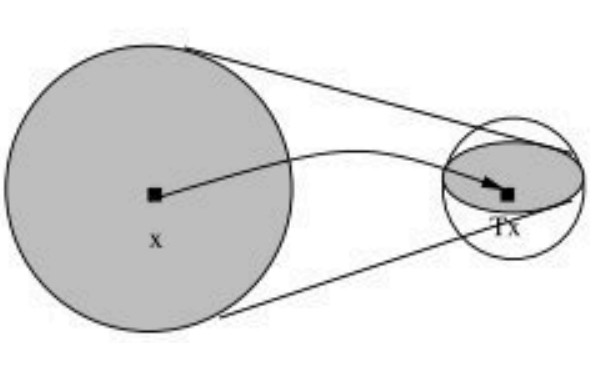


Software:
Matlab, Mathematica, Overleaf: WFU Software Link

OBJECTIVES



Develop Deep Understanding of Dynamical Systems

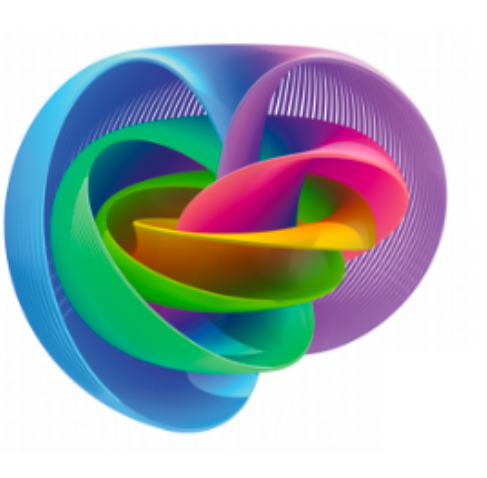


Develop Proficiency in Applied Analysis

Develop Mathematical Computational Skills

$$J_i = \begin{bmatrix} \lambda_i & 1 & 0 & \cdots & 0 \\ 0 & \lambda_i & 1 & & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_i & 1 \\ 0 & 0 & 0 & \cdots & \lambda_i \end{bmatrix}_{k \times k}$$

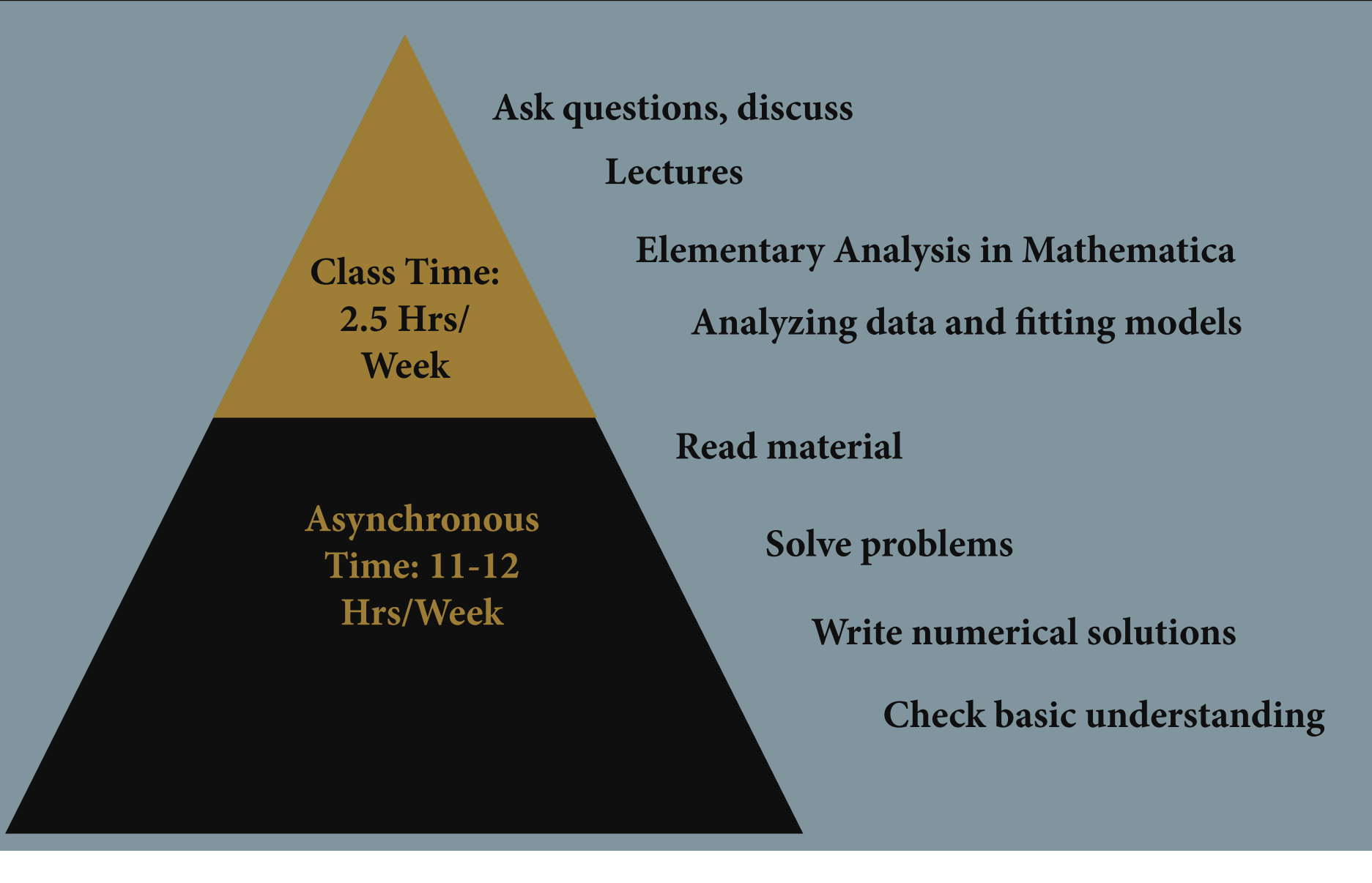
Master Linear Algebra



Develop Proficiency in Hamiltonian Dynamics

Write Effectively

CLASS STRUCTURE



EVALUATION

We focus on learning and mastery. You are guaranteed the following grades if your final percentage lies within the following ranges.

90-92.9: A-	93-100: A	
80-82.9: B-	83-86.9: B	87-89.9: B+
70-72.9: C-	73-76.9: C	77-79.9: C+
60-62.9: D-	63-66.9: D	67-69.9: D+

If you are taking this course pass/fail are guaranteed a passing grade if you earn a 70 or above.

Term paper (15%)
Individual projects, open book, open internet
Topic chosen from the text that is not covered in the course

Weekly Homework (25%), at least 1 dropped
Open book, collaboration allowed with citation
Homework is due on Friday in class

Two summative assessments (30%)
In class, closed notes

Final Exam (30%)
Comprehensive
In class, closed notes

Weekly Homework: Homework will be assigned most weeks on Thursday and will be due Friday in class the following week. Late homework will not be accepted unless there are extreme circumstances. However, I will drop at least one homework assignment from your grade. While you are allowed to collaborate with your colleagues, homework must consist of solutions that show all steps, be your own work and be written clearly using complete sentences as appropriate (see homework policy). All homework will be submitted in class on paper. I will not accept digital versions of your homework.

Term Paper: Every graduate student is required to write a term paper on a topic from the text that is not covered in the course. The paper must be written in LaTeX. Your textbook is written for a two semester course and has a wealth of interesting topics to pick from. In addition to writing an expository paper, you will be required to work through some of the problems from the text. The selection of the topic must be done in consultation with the faculty member.

Summative Assessments: There will be two in class summative assessments in the course and a cumulative final exam.

Late Work Policy: Except in very extreme circumstances, I do not accept late assignments or reschedule exams. If you have a situation in which you cannot make an exam for personal reasons, you must arrange accommodations with the instructor to schedule the exam **before the scheduled exam date.** If you have a legitimate emergency situation, I will make sure that all students in the course will have access to the same exception to this policy.

If you need to miss class due to a university sponsored activity, such as athletics. Please contact the faculty member as soon as possible to reschedule due dates.

COURSE ENVIRONMENT

Names/Pronouns

You **deserve** to be addressed in the manner you prefer. To guarantee that I address you properly, you are welcome to tell me your pronoun(s) and/or preferred name at any time, either in person or via email.

Diversity

We embrace diversity of age, background, beliefs, ethnicity, gender, gender identity, gender expression, national origin, religious affiliation, sexual orientation, and other visible and non-visible categories. **I do**

Accessibility

I want you to succeed in this course. Wake Forest University provides reasonable accommodations to students with disabilities. If you are in need of an accommodation, then please contact me privately as early in the term as possible. Retroactive accommodations may not be provided. Students requiring accommodations must also consult the Center for Learning, Access, and Student Success(118 Reynolda Hall, 336-758-5929, http://class.wfu.edu). For personal issues, stress, health problems or life circumstances see shs.wfu.edu/. Contact me if you have other special circumstances. **I will find resources for you.**

Title IX

You **deserve** a community free from discrimination, sexual harassment, a hostile environment, sexual assault, domestic violence, dating violence, and stalking. If you experience or know of a Title IX violation, you have many options for support and/or reporting; see titleix.wfu.edu/.

Emergency Fund

You **deserve** a learning environment in which all of your physiological and safety needs are met. If you are experiencing situations in which these needs are not met, e.g. you do not have adequate housing or sufficient food security, the Chaplain's Office has an emergency fund which can provide support: https://chaplain.wfu.edu/care-support/chaplains-emergency-fund/. In situations in which you need immediate assistance there is emergency funding available through the Department of Mathematics and Statistics. If you are in need of emergency help you are encouraged to reach out to a faculty member in the Department of Mathematics and Statistics who will work with the chair of the department to address your needs.

Course Resources

The department has a limited amount of funding for class materials. If you cannot afford class materials, you are encouraged to contact the chair of the department privately as early in the term as possible. Due to the limited amount of funds, students must exhaust all other sources of funding before applying to the department for assistance.

The Honor Code

At Wake Forest, we expect you to behave as honorable citizens of the class, the university, and the world as a whole. When you complete an assignment with your name on it, you are representing that everything you are turning in is your own work. That means that you do not copy from other students, textbooks, or websites. If at any time I become aware of cheating or plagiarism in this course, I will submit the information to the honor council.

TENTATIVE COURSE CALENDAR

Linear Theory and Existence and Uniqueness

Week 1: **Introductory Topics**
1. 1/10: Modeling with ODEs, and one dimensional dynamics. **Sections 1.1-1.3**
2. 1/12: Qualitative dynamics in simple 2-d systems. **Section 1.5**
3. 1/14: Matrix ODEs, **Section 2.1**

Week 2: **Linear Systems Part I**
1. 1/17: **MLK Day**
2. 1/19: **Snow Day (Nothing Useful Happened)**
3. 1/21: Two dimensional linear systems. **Section 2.2**

Week 3: **Linear Systems Part II**
1. 1/24: Exponentials of Operators. **Section 2.3**
2. 1/26: Complex eigenvalues and multiple eigenvalues, **Sections 2.5-2.6**
3. 1/28: Multiple Eigenvalues **Section 2.7**

Week 4: **Useless Week**
1. 1/31: Multiple Eigenvalues **Section 2.7**
2. 2/02: **Fertilizer Bomb**
3. 2/04: **Fertilizer Bomb**

Week 5: **Function Spaces**
1. 2/07: Multiple Eigenvalues **Section 2.7**
2. 2/09: Function Spaces **Section 3.1-3.2**
3. 2/11: Contraction Mapping Theorem **Section 3.2**

Week 6: **Existence and Uniqueness**
1. 2/14: Lipschitz Functions, **Section 3.2**
2. 2/16: **Summative Assessment on Weeks 1-5**
3. 2/18: Existence and Uniqueness and Gronwall's Inequality, **Sections 3.3-3.4**

Classical Dynamical Systems

Week 7: **Introduction to Flows**
1. 2/21: Flows and global existence, **Sections 4.1-4.3**
2. 2/23: Linearization and Stability, **Section 4.4-4.5**
3. 2/25: Lyapunov functions, **Sections 4.6**

Week 8: **Hartman-Grobman Theorem**
1. 2/28: Topological Conjugacy, **Sections 4.7**
2. 3/01: Hartman-Grobman Theorem, **Section 4.8**
3. 3/03: Omega limit sets, attractors, and basins of attraction, **Sections 4.9-4.10**

Spring Break: 3/5-3/13

Week 9: **Periodic Orbits and Poincare Maps**
1. 3/14: Stability of periodic orbits, **Section 4.11**
2. 3/16: Poincare maps, **Section 4.12**
3. 3/18: Stable and unstable sets, heteroclinic orbits, **Sections 5.1-5.2**

Week 10: **Stable Manifold Theorem**
1. 3/21: Stable manifold theorem, **Sections 5.3-5.4**
2. 3/23: Global stable manifolds, **Section 5.5**
3. 3/25: Center manifolds, **Section 5.6**

Week 11: **Center Manifolds and Non-Hyperbolic Fixed Points**
1. 3/28: Non-hyperbolic fixed points, **Section 6.1-6.3**
2. 3/30: **Summative Assessment on Weeks 6-10**
3. 4/01: Index theory, **Section 6.5**

Hamiltonian Dynamics

Week 12: **Hamiltonian Dynamics Part I**
1. 4/04: Conservative dynamics and volume preserving flow, **Sections 9.1-9.3**
2. 4/06: Poisson dynamics, **Section 9.4**
3. 4/08: The action principle, **Section 9.5**

Week 13: **Hamiltonian Dynamics Part II**
1. 4/11: Poincare Invariant, **Sections 9.6**
2. 4/13: Lagrangian Systems, **Section 9.7**
3. 4/15**: Good Friday**

Week 14: **Calculus of Variations**
1. 4/18: Calculus of Variations, **Section 9.8**
2. 4/20: Equivalence of Hamiltonian and Lagrangian Mechanics, **Section 9.9**
3. 4/22: Linearized Hamiltonian Systems, **Section 9.10**

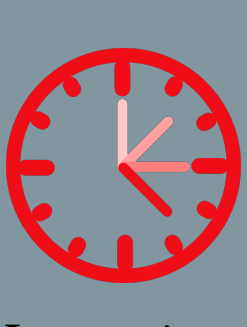
Week 15: **Integrability**
1. 4/25: Integrability and near integrability, **Sections 9.12**
2. 4/27: KAM Theory and chaos I
3. 4/29: KAM Theory and chaos II

Final Exam: Who Knows

SUCCESS

Attend class

Participate constantly



Invest time

Concentrate on concepts in addition to calculations

Seek help when needed

Eliminate Virtual Distractions