Probability
Spring 2023
Exam 1
02/10/23

Name (Print): Key

This exam contains 7 pages (including this cover page) and 8 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a "fundamental theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **You do not have to give explicit numbers for solutions to problems, i.e. you can leave the solution as a product and/or sum of numbers and you do not have to expand out terms with a factorial or binomial coefficients.**
- **Short answer questions:** Questions labeled as **"Short Answer"** can be answered by simply writing an equation or a sentence or appropriately drawing a figure. No calculations are necessary or expected for these problems.
- **Unless the question is specified as short answer, mysterious or unsupported answers might not receive full credit.** An incorrect answer supported by substantially correct calculations and explanations might receive partial credit.

Problem	Points	Score
1	10	
2	10	
3	15	
4	10	
5	10	
6	10	
7	15	
8	20	
Total:	100	

Do not write in the table to the right.

1. (10 points) (**Short Answer**) Determine if the following statement is correct (**C**) or incorrect (**I**). Just circle **C** or **I**. No need to show any work. In order for a statement to be correct it must be true in all cases.

   C **I** If A and B are events in a sample space S, then $P(A) = P(A \cap B) + P(A \cap \overline{B})$.

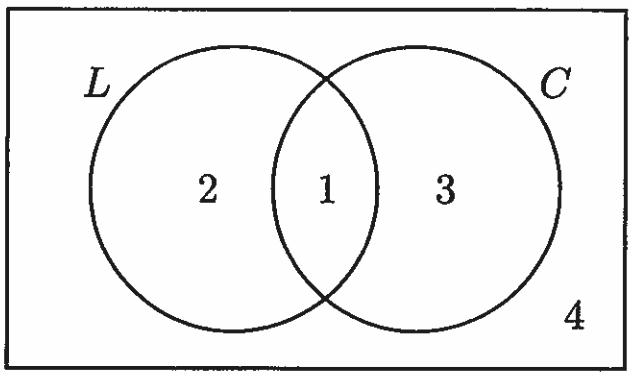
   C **I** If A and B are events in a sample space S, then $P(A|B) + P(B|A) = 1$

   C **I** If A, B, C are events in a sample space S such that $P(A) > P(B)$ and $P(C) > 0$, then $P(A|C) > P(B|C)$.

   C **I** If A and B are events in a sample space S, then $P(A|B) + P(\overline{A}|B) = 1$.

   C **I** If A and B are events in a sample space S, then $P(A|B) + P(A|\overline{B}) = 1$.

2. (10 points) **Short Answer:** In the Venn diagram below, L is the event that a driver has liability insurance and C is the event that they have collision insurance. Briefly, i.e., at most a sentence, express in words what events are represented by regions 1, 3, and 4.



1: The driver has liability and collision insurance.

~~2:~~

3: The driver has collision insurance but not liability insurance.

4. The drive has neither liability insurance nor collision insurance.

3. (15 points) At a local grocery store, packages of cocoa beans from the Amazon are inspected for sanitation. Suppose 5% are infested with spiders, 10% are moldy and 2% are both infested with spiders and are moldy.

(a) (3 points) What is the probability a randomly selected package is infested with spiders?

$P(I) = .05$

(b) (4 points) What is the probability a randomly selected package is infested with spiders or is moldy?

$$P(I \cup M) = P(I) + P(M) - P(I \cap M)$$
$$= .05 + .10 - .02$$
$$= .13$$

(c) (4 points) What is the the probability that a randomly selected package is infected with spiders but is not moldy?

$$P(I \cap \bar{M}) = P(I) - P(I \cap M)$$
$$= .05 - .02$$
$$= .03$$

(d) (4 points) What is the probability that a randomly selected package is neither moldy nor infected with spiders?

$$P(\bar{I} \cap \bar{M}) = P(\overline{I \cup M}) = 1 - P(I \cup M) = .87$$

4. (10 points) The price of a European tour includes four stopovers to be selected from among 10 cities. In how many different ways can one plan such a tour if

(a) (5 points) the order of the stopover matters;

$10 \cdot 9 \cdot 8 \cdot 7$ number of ways

(b) (5 points) the order of the stopovers does not matter.

$\binom{10}{4}$ number of ways

5. (10 points) How many different seven-digit telephone numbers can be formed if the first digit cannot be zero?

$9 \cdot 10^6$ = # of telephone numbers

6. (10 points) How many distinct permutations are there of the letters in the word "struts"?

$\frac{6!}{2!2!}$ = # of permutations.

7. (15 points) Suppose five of a company's 10 delivery trucks do not meet emission standards and three of them are chosen for inspection.

   (a) (10 points) What is the probability that none of the trucks chosen for inspection will meet emission standards?

$$\frac{1}{12} = \frac{5}{10} \cdot \frac{4}{9} \cdot \frac{3}{8} = \cancel{\text{\# of ways}} = \text{probability}$$

   (b) (5 points) What is the probability that at least one of the trucks chosen for inspection will not meet emission standards?

The probability that at least one fails is given by

$$1 - P(\text{all pass}) = 1 - \frac{5}{10} \cdot \frac{4}{9} \cdot \frac{3}{8} = \frac{11}{12}$$

8. (20 points) In a factory, machines I, II and III are all producing springs of the same length. Machine I produces 35% of the springs, machine II produces 25% of the springs, and machine III produces 40% of the springs. Machines I, II, and III produce 2%, 1%, and 3% defective springs, respectively.

(a) (10 points) What is the probability a defective spring is produced?

$$P(D) = P(D|I)P(I) + P(D|II)P(II) + P(D|III)P(III)$$
$$= .02 \cdot .35 + .01 \cdot .25 + .03 \cdot .40$$
$$= \frac{75+25+120}{10^4}$$
$$= \frac{220}{10^4}$$
$$= .022$$

(b) (10 points) If a defective spring was produced, what is the probability it came from machine II?

$$P(II|D) = \frac{P(D|II)P(II)}{P(D)}$$
$$= \frac{.01 \cdot .25}{.022}$$
$$= \frac{25}{220}$$
$$= 5/44$$