MTH 357/657
Homework #3

Due Date: February 03, 2022

1 Problems for Everyone

1. Two cards are randomly drawn from a deck of 52 playing cards. Find the probability that both cards will be greater than 3 and less than 8.

2. At Roanoke college it is known that 1/3 of the students live off campus. It is also known that 5/9 of the students are from within the state of Virginia and that 3/4 of the students are from out of state or live on campus. What is the probability that a student selected at random from Roanoke College is from out of state and lives on campus?

3. Show that following are true

   (a) $P(A|B) \geq 0$

   (b) $P(B|B) = 1$

   (c) If A_1 and A_2 are mutually exclusive then

   $$P(A_1 \cup A_2|B) = P(A_1|B) + P(A_2|B).$$

4. Show that if events A and B are independent, then

   (a) A and $\overline{B}$ are independent.

   (b) $\overline{A}$ and $\overline{B}$ are independent.

5. If events A, B, and C are independent, show that

   (a) A and $B \cap C$ are independent

   (b) A and $B \cup C$ are independent

6. A bin contains 100 balls, of which 25 are red, 40 are white, and 35 are black. If two balls are selected from the bin, what is the probability that one will be red and one will be white

   (a) if the first ball is replaced before the second is drawn;

   (b) if the second ball is drawn without replacing the first?

2 7. Suppose days can be either rainy or sunny and the probability that a rainy day is followed by a rainy day is .80 and the probability that a sunny day is followed by a rainy day is .60. Find the probabilities that a rainy day is followed by

    (a) a rainy day, a sunny day, and another rainy day;

    (b) two sunny days and then a rainy day;

    (c) two rainy days and then two sunny days;

    (d) rain two days later.

8. A coin is loaded so that the probabilities of heads and tails are .52 and .48 respectively. If the coin is tossed three times, what are the probabilities of getting

    (a) all heads;

    (b) two tails and a head in that order.

9. A shipment of 1000 parts contains 1 percent defective parts. Find the probability that

    (a) the first four parts chosen arbitrarily for inspection are not defective;

    (b) the first defective part found will be on the fourth inspection.

10. Medical records show that one out of 10 persons in a certain town has a thyroid deficiency. If 12 persons in this town are randomly chosen and tested, what is the probability that at least one of them will have a thyroid deficiency?

2 11. A mail-order house employs three stock clerks, U, V, and W, who pull items from shelves and assemble them for subsequent verification and packaging. U makes a mistake in an order (gets a wrong item or the wrong quantity) one time in a hundred, V makes a mistake in an order five times in a hundred, and W makes a mistake in an order three times in a hundred. If U, V, and W, fill, respectively, 30, 40, and 30 percent of all orders, what are the probabilities that

    (a) a mistake will be made in an order;

    (b) if a mistake is made in an order, the order was filled by U;

    (c) if a mistake is made in an order, the order was filled by V?

Homework #3

#1.

Two cards are randomly drawn from a deck of 52 playing cards. Find the probability that both cards will be greater than 3 and less than 8.

Solution:

There are four cards between 3 and 8 and thus the probability is given by
$$p = \frac{4 \cdot 4}{52} \cdot \frac{4 \cdot 4 - 1}{51} = .09.$$

#2.

Show that the following are true
(a) $P(A|B) \geq 0$
(b) $P(B|B) = 1$
(c) If A_1 and A_2 are mutually exclusive then
$$P(A_1 \cup A_2 | B) = P(A_1 | B) + P(A_2 | B).$$

Solution:

(a) Since $P(A \cap B) \geq 0$ and $P(B) > 0$ it follows that
$$P(A|B) = \frac{P(A \cap B)}{P(B)} \geq 0.$$

(b). Computing we have that
$$P(B|B) = \frac{P(B)}{P(B)} = 1.$$

(c). Since A_1, A_2 are mutually exclusive we have that
$$P(A_1 \cup A_2 | B) = \frac{P((A_1 \cup A_2) \cap B)}{P(B)} = \frac{P((A_1 \cap B) \cup (A_2 \cap B))}{P(B)}$$

$$\Rightarrow P(A_1 \cup A_2 \mid B) = \frac{P(A_1 \cap B)}{P(B)} + \frac{P(A_2 \cap B)}{P(B)} = P(A_1 \mid B) + P(A_2 \mid B).$$

#4.

Show that if A and B are independent, then

(a) A and $\overline{B}$ are independent.

(b) $\overline{A}$ and $\overline{B}$ are independent.

Solution:

(a) Computing we have that

$$P(A) = P((A \cap B) \cup (A \cap \overline{B})) = P(A \cap B) + P(A \cap \overline{B}) = P(A)P(B) + P(A \cap \overline{B})$$

$$\Rightarrow P(A)(1 - P(B)) = P(A \cap \overline{B})$$

$$\Rightarrow P(A)P(\overline{B}) = P(A \cap \overline{B}).$$

(b) Computing we have that

$$P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B})$$

$$= 1 - P(A \cup B)$$

$$= 1 - P(A) - P(B) + P(A \cap B)$$

$$= 1 - P(A) - P(B) + P(A)P(B)$$

$$= P(\overline{A}) - P(B)(1 - P(A))$$

$$= P(\overline{A}) - P(B)P(\overline{A})$$

$$= P(\overline{A})(1 - P(B))$$

$$= P(\overline{A})P(\overline{B}).$$

#7

Suppose days can be either rainy or sunny and the probability that a rainy is followed by a rainy day is .80 and the probability that a sunny day is followed by a rainy day is .60. Find the probabilities that a rainy day is followed by

(a) A rainy day, a sunny day, and another rainy day.

(b) Two sunny days and then a rainy day.

(c) Two rainy days and then two sunny days.

(d) Rain two days later.

Solution

Let R and S denote probabilities of rainy or sunny days respectively. Therefore,

$$P(R|R) = .8$$
$$P(R|S) = .6$$

and thus

$$P(S|R) = 1 - P(R|R) = .2 .$$
$$P(S|S) = 1 - P(R|S) = .4$$

.8·.2·.6 .096 (a) $P(R|R) \cdot P(S|R) \cdot P(R|R) = .8 \cdot .2 \cdot .6 = .064$

.048 (b) $P(S|R) P(S|S) \cdot P(R|S) = .048$

.12 + .64 .76

.8·.8·.2·.4=.0512 (c) $P(S|R) P(R|S) + P(R|R) \cdot P(R|R) = .16 + .64 = .8$

.76 ↵

#11

A mail-order house employs three stock clerks U, V, and W, who pull items from shelves and assemble them for subsequent verification and packaging. U makes a mistake one time in a hundred, V five times in a hundred, W three times in a hundred. If U, V, and W, fill, respectively, 30, 40, and 30 percent of all orders, what are the probabilities that

(a) A mistake will be made.

(b) if a mistake is made, the order was filled by U

(c) if a mistake is made, the order was filled by V.

Solution:

(a) $P(M) = P(M|U)P(U) + P(M|V)P(V) + P(M|W)P(W)$

$= .01 \cdot .3 + .05 \cdot .4 + .03 \cdot .3$

$= .032$

(b) $P(U|M) = \dfrac{P(M|U)P(U)}{.032}$

$= \dfrac{.01 \cdot .3}{.032}$

$= .09375$

(c) $P(V|M) = \dfrac{P(M|V)P(V)}{.032}$

$= \dfrac{.05 \cdot .4}{.032}$

$= .625$