MTH 357/657
Homework #9

Due Date: April 07, 2023

1 Discrete Multivariate Distributions

1. The joint probability distribution $p(x, y)$ of random random variables X and Y satisfies

$$
\begin{aligned}
&p(0,0) = \frac{1}{12},\ p(1,0) = \frac{1}{6},\ p(2,0) = \frac{1}{24},\\
&p(0,1) = \frac{1}{4},\ p(1,1) = \frac{1}{4},\ p(2,1) = \frac{1}{40},\\
&p(0,2) = \frac{1}{8},\ p(1,2) = \frac{1}{20},\\
&p(0,3) = \frac{1}{120}.
\end{aligned}
$$

   (a) Find the following values:
      i. $P(X = 1, Y = 2)$
      ii. $P(X = 0, 1 \leq Y < 3)$
      iii. $P(X + Y \leq 1)$
      iv. $P(X > Y)$

   (b) If $F(x, y)$ denotes the joint cumulative distribution function for this probability distribution, find the following values:
      i. $F(1.2, .9)$
      ii. $F(-3, 1.5)$
      iii. $F(2, 0)$
      iv. $F(4, 2.7)$

2. Suppose the joint probability distribution function of X and Y is given by

$$f(x, y) = c(x^2 + y^2)$$

   for $x = -1, 0, 1, 3$ and $y = -1, 2, 3$ and zero otherwise.

   (a) Find the value of c.
   (b) Compute $P(X \leq 1, Y > 2)$
   (c) Compute $P(X = 0, Y \leq 2)$
   (d) Compute $P(X + Y > 2)$

3. Show that there is no value of k for which

$$f(x, y) = ky(2y - x),\ \text{for } x = 0, 1, 2, 3 \text{ and } y = 0, 1, 2$$

   can serve as the joint probability distribution of two random variables.

2 Continuous Multivariate Distributions

1. If a radioactive particle is randomly located in a square of unit length, with coordinates (X, Y) selected randomly. A reasonable model for the joint density probability function for its X and Y is

$$f(x,y) = \begin{cases} 1 & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & \text{elsewhere} \end{cases}.$$

   (a) What is $P(X > .5, Y > .5)$?

   (b) What is $P(X - Y > .5)$?

   (c) What is $P(XY < 1/2)$?

2. Determine k so that

$$p(x,y) = \begin{cases} kx(x-y) & 0 < x < 1,\ -x < y < x \\ 0 & \text{elsewhere} \end{cases}$$

   can serve as a joint probability density.

3. If the joint probability density of X and Y is given by

$$p(x,y) = \begin{cases} 24xy & 0 < x < 1,\ 0 < y < 1,\ x + y < 1 \\ 0 & \text{elsewhere} \end{cases},$$

   find $P\left(X + Y < \frac{1}{2}\right)$.

4. If the joint probability density of X and Y is given by

$$f(x,y) = \begin{cases} 2 & \text{for } x > 0,\ y > 0,\ x + y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

   find

   (a) $P(X \le \frac{1}{2}, Y \le \frac{1}{2})$;

   (b) $P(X + Y > \frac{2}{3})$;

   (c) $P(X > 2Y)$.

5. pg. 234, #5.11.

3 Marginal and Conditional Distributions

1. With reference to problem #1 in the "Discrete Multivariate Distributions" section of this assignment, find

   (a) The marginal distribution of X;

   (b) The marginal distribution of Y;

   (c) The conditional distribution of X given $Y = 1$;

   (d) The conditional distribution of Y given $X = 0$;

2. Check whether the discrete random variables X and Y are independent if their joint probability distribution $f(x, y)$ is given by

   (a) $f(x, y) = 1/4$ for $x = -1$ and $y = -1$, $x = -1$ and $y = 1$, $x = 1$ and $y = -1$, and $x = 1$ and $y = 1$;

   (b) $f(x, y) = 1/3$ for $x = 0$ and $y = 0$, $x = 0$ and $y = 1$, and $x = 1$ and $y = 1$.

3. If the joint probability density of X and Y is given by

$$f(x, y) = \begin{cases} \frac{1}{4}(2x + y) & \text{for } 0 < x < 1, 0 < y < 2 \\ 0 & \text{elsewhere} \end{cases}$$

   find

   (a) the marginal density of X;

   (b) the conditional density of Y given $X = 1/4$;

   (c) the marginal density of Y;

   (d) the conditional density of X given $Y = 1$.

4. If the independent random variables X and Y have the marginal densities

$$f(x) = \begin{cases} \frac{1}{2} & \text{for } 0 < x < 2 \\ 0 & \text{elsewhere} \end{cases}$$

$$g(y) = \begin{cases} \frac{1}{3} & \text{for } 0 < y < 3 \\ 0 & \text{elsewhere} \end{cases}$$

   (a) Find the joint probability density of X and Y.

   (b) Find $P(X^2 + Y^2 > 1)$.

5. Suppose the joint density of X and Y is given by

$$f(x, y) = \begin{cases} \frac{1}{y} & \text{for } 0 < x < y,\ 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

   (a) Find the probability that the sum of X and Y will exceed 1/2.

   (b) Find the marginal density of X.

   (c) Find the marginal density of Y.

   (d) Determine whether the two random variables are independent.