Lecture #1: Sample Spaces and Events

Experiment: Any process of observation or measurement.

Outcome: Results one obtains from an experiment

Sample Space: Set of all possible outcomes and usually denoted by S. Each element of the sample space is called an element, sample point, or simple event.

Examples:

1. Flipping a coin
   $\Rightarrow S = \{\text{Heads, Tails}\}$
2. Set of automobiles with diesel engines
   $\Rightarrow S = \{c : c \text{ is an automobile with diesel engine}\}$
3. Set of positive odd integers
   $\Rightarrow S = \{2n+1 : n = 0, 1, 2, \ldots\}$
4. Roll a die a record number
   $\Rightarrow S_1 = \{1, 2, 3, 4, 5, 6\}$
5. Roll a dice and record even or odd
   $\Rightarrow S_2 = \{\text{even, odd}\}$.

*Note: It is preferable that we use the largest sample space

6. Roll a pair of dice, one red and one green
   $\Rightarrow S = \{(r, g) : r = 1, \ldots, 6; g = 1, \ldots, 6\}$.

7. Flip a coin until ~~→~~ get a heads
$S = \{H, TH, TTH, TTTH, \ldots\}$

Events: Subsets of a sample space.

Examples

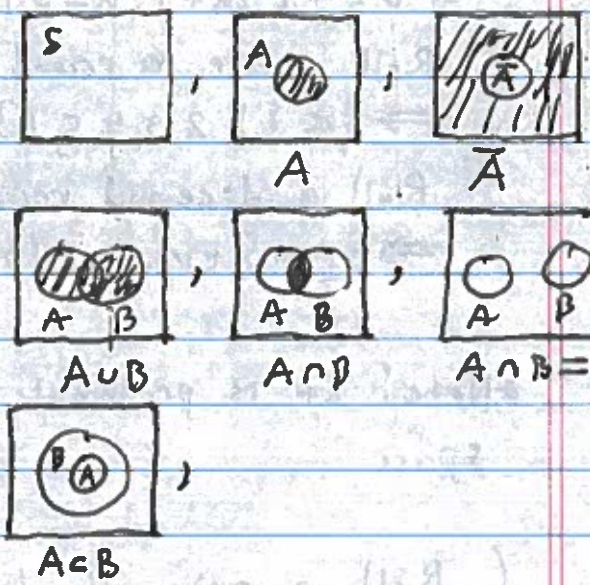
$S_1 = \{1, 2, 3, 4, 5, 6\}$ → outcome of rolling one die

$S_2 = \{(r, g) : r = 1, \ldots, 6,\ g = 1, \ldots, 6\}$ → outcome of rolling red/green dice

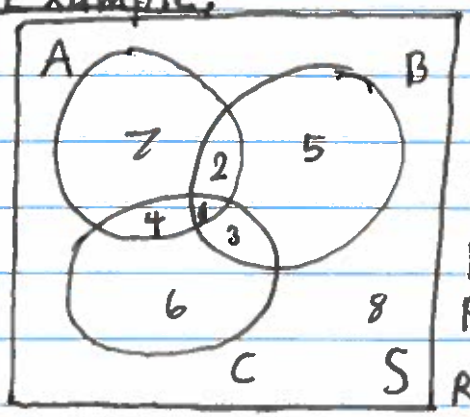
1. A = event that I roll is divisible by 3
   $= \{3, 6\}$
2. B = event that sum of two dice is 8
   $= \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$.
3. Someone takes three shots at a target and we only record a hit or miss. Let 0 and 1 denote miss and hit respectively.
   $\Rightarrow S = \{(0,0,0), (1,0,0), (0,1,0), (0,0,1), (1,1,0), (1,0,1), (0,1,1), (1,1,1)\}$.

Venn Diagrams:

- S = sample space
- $A, B \subset S$ are events
- $\bar{A} = \{x \in S : x \notin A\}$
- $A \cup B = \{x \in S : x \in A \text{ or } x \in B\}$
- $A \cap B = \{x \in S : x \in A \text{ and } x \in B\}$



Example:



$R1 = A \cap B \cap C$

$R2 = A \cap B \cap \bar{C}$

$R3 = B \cap C \cap \bar{A}$

$R4 = A \cap C \cap \bar{B}$

$R5 = B \cap \bar{C} \cap \bar{A}$

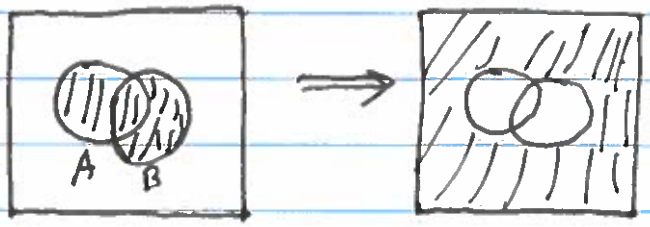
$R6 = C \cap \bar{A} \cap \bar{B}$

$R7 = A \cap \bar{B} \cap \bar{C}$

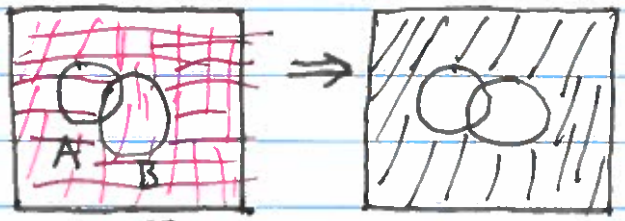
$R8 = \bar{A} \cap \bar{B} \cap \bar{C}$

Example:

$\overline{A \cup B} = \bar{A} \cap \bar{B}$



$A \cup B$ $\qquad$ $\overline{A \cup B}$



$\bar{A}, \bar{B}$ $\qquad$ $\bar{A} \cap \bar{B}$