Lecture #11: Hypergeometric Distribution

Among 120 applicants only 80 are qualified. If five applicants are randomly selected for an interview, find the probability that two of the five are qualified.

Three approaches:

1. $p(QQ\bar{Q}\bar{Q}\bar{Q}) = \frac{80}{120} \cdot \frac{79}{119} \cdot \frac{40}{118} \cdot \frac{39}{117} \cdot \frac{38}{116}$

   For other combinations, the denominator is the same but the numerator is rearranged

   $\Rightarrow p(2) = \binom{5}{2} \frac{80!}{78!} \frac{40!}{37!} \frac{115!}{120!} = .1638$

2. $p(2) = \dfrac{\binom{80}{2}\binom{40}{3}}{\binom{120}{5}} = .1638$

3. Approximate with binomial distribution with probability of success $p = 80/120 = 2/3$

   $\Rightarrow p(2) = \binom{5}{2}\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^5 = .1646.$

Definition - A random variable is said to have a hypergeometric distribution if and only if

$$p(x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$$

$\rightarrow$ N objects, r of certain type, probability of x selected of type r when picking n.