

Lecture 15: Cumulative Distribution Functions

Definition - If X is a discrete random variable, the function given by

$$F(x) = P(X \leq x) = \sum_{t \leq x} f(t),$$

where $f(t)$ is the probability distribution of X at t , is called the distribution function or the cumulative distribution function (CDF).

Properties

1. $\lim_{x \rightarrow -\infty} F(x) = 0$
2. $\lim_{x \rightarrow \infty} F(x) = 1$
3. If $a < b$, then $F(a) \leq F(b)$ (monotone increasing)

Example:

Find the CDF for the number of heads in 4 coin tosses.

$$f(0) = \binom{4}{0} \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

$$f(1) = \binom{4}{1} \left(\frac{1}{2}\right)^4 = \frac{4}{16}$$

$$f(2) = \binom{4}{2} \left(\frac{1}{2}\right)^4 = \frac{6}{16}$$

$$f(3) = \binom{4}{3} \left(\frac{1}{2}\right)^4 = \frac{4}{16}$$

$$f(4) = \binom{4}{4} \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

$$\Rightarrow F(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{16}, & 0 \leq x < 1 \\ \frac{5}{16}, & 1 \leq x < 2 \\ \frac{11}{16}, & 2 \leq x < 3 \\ \frac{15}{16}, & 3 \leq x < 4 \\ 1, & 4 \leq x \end{cases}$$

