Lecture 5: Binomial Theorem

$$(x+y)^2=(x+y)(x+y)=x^2+xy+yx+y^2=x^2+2xy+y^2$$

$$(x+y)^3=(x+y)(x+y)(x+y)=x^3+xxy+xyx+xyy+yxx+yxy$$
$$+yyx+y^3$$
$$=x^3+3x^2y+3xy^2+y^3$$
$$=\binom{3}{0}x^3+\binom{3}{1}x^2y+\binom{3}{2}xy^2+\binom{3}{3}y^3$$

Theorem - $(x+y)^n=\binom{n}{0}x^n+\binom{n}{1}x^{n-1}y^1+\dots+\binom{n}{n-1}x^1y^{n-1}+\binom{n}{n}y^n$

$$=\sum_{r=0}^{n}\binom{n}{r}x^n y^{n-r}$$

Pascal's Triangle:

0 - 1

1 - 1 1

2 - 1 2 1

3 - 1 3 3 1

4 - 1 4 6 4 1

$$6=\binom{4}{2}$$

Theorem -

$$\binom{n}{r}=\binom{n-1}{r}+\binom{n-1}{r-1}$$

proof

$$(1+y)^n=(1+y)(1+y)^{n-1}$$

$$\sum_{r=0}^{n}\binom{n}{r}y^r=(1+y)^{n-1}+y(1+y)^{n-1}$$

$$=\sum_{r=0}^{n-1}\binom{n-1}{r}y^r+y\sum_{r=0}^{n-1}\binom{n-1}{r}y^r$$

$$=\sum_{r=0}^{n-1}\binom{n-1}{r}y^r+\sum_{r=0}^{n-1}\binom{n-1}{r}y^{r+1}$$

$$\Rightarrow \sum_{r=0}^{n} \binom{n}{r} y^r = \sum_{r=0}^{n-1} \binom{n-1}{r} y^r + \sum_{r=1}^{n} \binom{n-1}{r-1} y^r$$

$$\Rightarrow \binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}.$$