

Key

MTH 357/657

Quiz #7

1. Suppose  $X$  is a continuous random variable with probability density function

$$p(x) = \begin{cases} kx^{\frac{1}{3}} & 0 < x < 8 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Find the value of  $k$ .

$$\int_0^8 kx^{\frac{1}{3}} dx = k \cdot \frac{3}{4} x^{\frac{4}{3}} \Big|_0^8 = k \cdot \frac{3}{4} 8^{\frac{4}{3}} = k \cdot \frac{3}{4} \cdot 16 = 12k$$
$$\Rightarrow k = \frac{1}{12}$$

(b) Find the cumulative distribution function for this random variable.

$$F(x) = P(X \leq x)$$
$$= \begin{cases} 0, & x < 0 \\ \frac{1}{12} \int_0^x x^{\frac{1}{3}} dx, & 0 \leq x \leq 8 \\ 1, & x > 8 \end{cases}$$
$$= \begin{cases} 0, & x < 0 \\ \frac{1}{16} x^{\frac{4}{3}}, & 0 \leq x \leq 8 \\ 1, & x > 8 \end{cases}$$