

MTH 352/652

Quiz #6

1. Construct a finite difference approximation to the second derivative of a function  $f(x)$  using its values at the points  $x$ ,  $x + \Delta x$  and  $x - 2\Delta x$ .

$$f(x + \Delta x) = f(x) + f'(x)\Delta x + \frac{f''(x)}{2!}\Delta x^2 + \frac{f'''(x)}{3!}\Delta x^3 + \dots$$

$$f(x - 2\Delta x) = f(x) - 2f'(x)\Delta x + 2\frac{f''(x)}{2!}\Delta x^2 - \frac{f'''(x)}{3!}\Delta x^3 + \dots$$

$$\Rightarrow 2f(x + \Delta x) + f(x - 2\Delta x) = 3f(x) + \frac{1}{3}f''(x)\Delta x^2 + O(\Delta x^3)$$

$$\Rightarrow f''(x) = \frac{2f(x + \Delta x) - 3f(x) + f(x - 2\Delta x)}{\Delta x^2} + O(\Delta x)$$

2. What is the order of this approximation?

$$O(\Delta x)$$