MTH 352/652
Quiz #7

Special Directions: This is a small group quiz with the teams assigned below. You are only allowed to work with your teammates. Any intentional communication with other teams will result in a zero for your entire team. Each member of the team will receive the same grade based on the lowest score.

Team #1	Team #2	Team #3	Team #4
Emily	Lyle	Sarah	Ethan
Jasmine	Yiyi	Shelby	Mandy
Nick	Cordell	Jiachen	Colin
Miguel	Cole	Steven	Clary

1. Consider heat flow in a rod of length L where the heat loss across the lateral boundary is given by Newton's law of cooling. The model is

$$u_t = ku_{xx} - hu,$$
$$u(t, 0) = 0,$$
$$u(t, L) = 0,$$
$$u(0, x) = f(x),$$

where $k, h > 0$.

(a) Write down the equation satisfied by the state steady solution for this problem.

$$Ku^*_{xx} - hu^*$$
$$\Rightarrow U^*_{xx} = \frac{h}{k}U^*$$

(b) Find the steady state solution for this problem. **Hint:** Hyperbolic trig functions might be helpful.

$$U^*(x) = A\cosh\left(\sqrt{\tfrac{h}{k}}x\right) + B\sinh\left(\sqrt{\tfrac{h}{k}}x\right)$$
$$U^*(0) = 0 = A \Rightarrow A = 0.$$
$$U^*(L) = B\sinh\left(\sqrt{\tfrac{h}{k}}L\right) = 0 \Rightarrow B = 0$$
$$\Rightarrow U^*(x) = 0$$

1