MTH 352/652
Classwork #1

1. Suppose $z = e^{x(s,t)} \sin(y(s,t))$ and $x(s,t) = st^2$, $y(s,t) = s^2 t$.

   (a) Directly substitute in $x(s,t)$ and $y(s,t)$ and then compute $\dfrac{\partial z}{\partial s}$.

   (b) Compute $\dfrac{\partial z}{\partial s}$ using the chain chain rule and then substitute in $x(s,t)$ and $y(s,t)$.

2. If $g(s,t) = f(s^2 - t^2, t^2 - s^2)$ and f is differentiable, show that g satisfies the following equation:

$$t\frac{\partial g}{\partial s} + s\frac{\partial g}{\partial t} = 0.$$

3. Suppose $z = f(x, y)$ has continuous second-order partial derivatives and $x = r^2 + s^2$ and $y = 2rs$.

   (a) Find $\dfrac{\partial z}{\partial r}$

   (b) Find $\dfrac{\partial^2 z}{\partial r^2}$

4. Suppose $z = f(x, y)$, where $x = g(s, t)$ and $y = h(s, t)$. Show that

$$\frac{\partial^2 z}{\partial t^2} = \frac{\partial^2 z}{\partial x^2}\left(\frac{\partial x}{\partial t}\right)^2 + 2\frac{\partial^2 z}{\partial x \partial y}\frac{\partial x}{\partial t}\frac{\partial y}{\partial t} + \frac{\partial^2 z}{\partial y^2}\left(\frac{\partial y}{\partial t}\right)^2 + \frac{\partial z}{\partial x}\frac{\partial^2 x}{\partial t^2} + \frac{\partial z}{\partial y}\frac{\partial^2 y}{\partial t^2}.$$