MTH 225
Quiz #6

1. The matrices $I, \sigma_x, \sigma_y, \sigma_z \in M_{2\times 2}(\mathbb{C})$ given by

$$I = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\ \sigma_x = \frac{1}{\sqrt{2}}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},\ \sigma_y = \frac{1}{\sqrt{2}}\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix},\ \sigma_z = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix},$$

are orthonormal with respect to the inner product $\langle \cdot, \cdot \rangle$ defined by

$$\langle A, B\rangle = \operatorname{tr}(A^* B).$$

Using inner products, find the coordinates of the matrix

$$A = \begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$$

with respect to the basis $\beta = \{I, \sigma_x, \sigma_y, \sigma_z\}$.

$\langle A, I\rangle = tr\left(\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\right) = \frac{2}{\sqrt{2}} = \sqrt{2}$

$\langle A, \sigma_x\rangle = tr\left(\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\right) = \frac{2}{\sqrt{2}} = \sqrt{2}$

$\langle A, \sigma_y\rangle = tr\left(\frac{1}{\sqrt{2}}\begin{bmatrix}i & -i\\ i & -i\end{bmatrix}\right) = 0$

$\langle A, \sigma_z\rangle = tr\left(\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -1\\ 1 & -1\end{bmatrix}\right) = 0$

$$\Rightarrow [A]_\beta = \begin{bmatrix}\sqrt{2}\\ \sqrt{2}\\ 0\\ 0\end{bmatrix}$$