

MTH 225

Quiz #7

1. **Short Answer:** Let $A \in M_{n \times n}(\mathbb{C})$. Write down at least two properties that A must satisfy in order for it to be a unitary matrix.

1. Columns of A are orthonormal

2. $A^* = A^{-1}$

3. $\langle A\vec{v}, A\vec{w} \rangle = \langle \vec{v}, \vec{w} \rangle$.

2. **Short Answer:** Let $A \in M_{n \times n}(\mathbb{C})$. Write down the definition of what it means for A to be a Hermitian matrix.

$$A^* = A \quad \text{or} \quad \langle A\vec{v}, \vec{w} \rangle = \langle \vec{v}, A\vec{w} \rangle$$

3. Show that if $A \in M_{n \times n}(\mathbb{C})$ is unitary and $A^2 = I$ then A is Hermitian.

If $A^2 = I$ then

$$A^* A \cdot A = A^*$$

$$\Rightarrow A = A^*$$