MTH 225
Quiz #8

1. Let $A \in M_{2\times 2}(\mathbb{C})$. Suppose that

$$\ker A = \operatorname{span}\left\{\begin{bmatrix}1\\3\end{bmatrix}\right\}, \quad \operatorname{im}(A) = \operatorname{span}\left\{\begin{bmatrix}1\\1\end{bmatrix}\right\}, \quad \text{and } \sigma_1 = \pi,$$

where σ_1 denotes the largest singular value of A. Find a possible singular value decomposition of A. That is, find unitary $U, V \in M_{2\times 2}(\mathbb{C})$ and diagonal $\Sigma \in M_{2\times}(\mathbb{R})$ such that

$$A = U\Sigma V^*.$$

$\vec{v}_2 = \frac{1}{\sqrt{10}}\begin{bmatrix}1\\3\end{bmatrix}$, $\sigma_2 = 0$

$\vec{v}_1 = \frac{1}{\sqrt{10}}\begin{bmatrix}3\\-1\end{bmatrix}$

$\vec{u}_2 = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$

$\vec{u}_1 = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$

$$\Rightarrow A = U\Sigma V^* = \begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}\begin{bmatrix}\pi & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\frac{3}{\sqrt{10}} & -\frac{1}{\sqrt{10}}\\ \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}}\end{bmatrix}$$