MTH 352/652
Homework #9

Due Date: April 11, 2025

1. Consider the following initial-boundary value problem:

$$\begin{aligned} &u_t = u_{xx}, \\ &u(0,t) = 1, \\ &u(2\pi,t) = 2, \\ &u(x,0) = 1. \end{aligned}$$

   (a) Calculate the steady state solution for this initial-boundary value problem.

   (b) Solve this initial boundary value problem.

2. Consider the following initial-boundary value problem for $x \in [0,1]$:

$$\begin{aligned} &u_t = u_{xx} - u, \\ &u(0,t) \\ &u(1,t) = 0, \\ &u(x,0) = \sin(3\pi x). \end{aligned}$$

   (a) Using separation of variables solve this initial-boundary value problem.

   (b) Using your solution, calculate $\lim_{t\to\infty} u(x,t)$.

   (c) Are there any steady state solutions to this equation? If so, what are they?

3. Consider the following initial-boundary value problem for $x \in [0,\pi]$:

$$\begin{aligned} &u_{tt} = u_{xx}, \\ &u_x(0,t) = 0 \\ &u_x(\pi,t) = 0, \\ &u(x,0) = \cos^2(x), \\ &u_t(x,0) = \cos(3x). \end{aligned}$$

   (a) Solve this initial-boundary value problem. **Hint:** It might be useful to use trig identities to reduce $\cos^2(x)$.

   (b) Sketch the solution for $t = 0$, $t = \pi/2$, $t = \pi$, $t = 3\pi/2$, and $t = 2\pi$.

   (c) Describe qualitatively the behavior of the solution.

4. Solve the following initial-boundary value problem for $x \in [0, \pi]$:

$$\begin{aligned} &u_{tt} + u_t = u_{xx}, \\ &u(0,t) = 0 \\ &u(\pi,t) = 0, \\ &u(x,0) = \sin^2(x), \\ &u_t(x,0) = 0. \end{aligned}$$

5. Solve the following boundary value problem on the domain $\Omega = [0,1] \times [0,1]$

$$\begin{aligned} \Delta u &= 0 \\ u(0,y) &= \sin(\pi y) \\ u(1,y) &= \sin(2\pi y) \\ u(x,0) &= \sin(3\pi x) \\ u(x,1) &= \sin(4\pi x). \end{aligned}$$