MTH 352
Quiz #7

1. For $L > 0$, the following partial differential equation models the heat flow in a pipe of length L:

$$u_t = u_{xx},$$
$$u_x(0,t) = 0,$$
$$u(L,t) = 0.$$

Note, I did not provide any initial conditions for this problem as it is not relevant to parts (a) and (b) below.

(a) **Short Answer:** Briefly interpret what the boundary conditions tell you about the heat at the boundary of the domain.

The flux is zero at $x=0$ and the heat is held constant at $u=0$ for $x=L$.

(b) By assuming a solution of the form $u(x,t) = X(x)T(t)$, find all separable solutions to this boundary value problem.

$$XT' = X''T$$
$$\Rightarrow \frac{X''}{X} = \frac{T'}{T} = -\lambda$$
$$\Rightarrow X = A\cos(\sqrt{-\lambda}x) + B\sin(\sqrt{-\lambda}x)$$
$$X'(0) = 0 \Rightarrow B = 0$$
$$X(L) = 0 \Rightarrow A\cos(\sqrt{-\lambda}L) = 0$$
$$\Rightarrow \sqrt{-\lambda}L = \frac{(2n-1)\pi}{2}$$
$$\Rightarrow \lambda = -(2n-1)^2\pi^2/4$$

Therefore,

$$u_n(x,t) = e^{-(2n-1)^2\pi^2 t/4}\cos\left(\frac{(2n-1)\pi}{2}x\right)$$