

Reminders:

## Exam preparation advice

- a. Prepare equation sheet
  - Basic equations used to solve problems
  - Should be able to derive equations from fundamental laws and definitions
- b. Review Homework problems
- c. Work as many problems as you can from old exams and other sources
- d. Get all of your questions answered
- e. Don't panic! (Most of you will be able to drop your lowest exam.)

## Introduction of the notion of Kinetic energy

Derivation:

Consider Newton's second law:

$$\begin{aligned}\mathbf{F}_{\text{total}} &= m \mathbf{a} \quad \Rightarrow \quad \mathbf{F}_{\text{total}} \cdot d\mathbf{r} = m \mathbf{a} \cdot d\mathbf{r} \\ \int_{\mathbf{r}_i}^{\mathbf{r}_f} \mathbf{F}_{\text{total}} \cdot d\mathbf{r} &= \int_{\mathbf{r}_i}^{\mathbf{r}_f} m \mathbf{a} \cdot d\mathbf{r} = \int_{\mathbf{r}_i}^{\mathbf{r}_f} m \frac{d\mathbf{v}}{dt} \cdot d\mathbf{r} = \int_{t_i}^{t_f} m \frac{d\mathbf{v}}{dt} \cdot \frac{d\mathbf{r}}{dt} dt = \int_{t_i}^{t_f} m \frac{d\mathbf{v}}{dt} \cdot \mathbf{v} dt \\ \int_{t_i}^{t_f} m \frac{d\mathbf{v}}{dt} \cdot \mathbf{v} dt &= \int_{\mathbf{v}_i}^{\mathbf{v}_f} m d\mathbf{v} \cdot \mathbf{v} = \int_i^f d\left(\frac{1}{2} m \mathbf{v} \cdot \mathbf{v}\right) = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2\end{aligned}$$

$$\Rightarrow W_{\text{total}} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

Kinetic energy (joules)

Kinetic energy:  $K = \frac{1}{2} m v^2$

$$\text{units: } (\text{kg}) (\text{m/s})^2 = \underbrace{(\text{kg m/s}^2)}_{\text{N}} \underbrace{\text{m}}_{\text{m}} = \text{joules}$$

Work – kinetic energy relation:

$$W_{\text{total}} = K_{\text{f}} - K_{\text{i}}$$

## On line quiz

Suppose you hit a golf ball having a mass of 0.4 kg with an initial velocity of  $v_i = 150$  m/s and an initial angle of 45 deg. Assume that the ball makes a perfectly parabolic trajectory and lands at the same vertical height as it started.

1. What is the initial kinetic energy of the ball?

- (a) -2250 J (b) 2250 J (c) -4500 J (d) 4500 J

2. What is the final kinetic energy of the ball just before it reaches the ground?

- (a) -2250 J (b) 2250 J (c) -4500 J (d) 4500 J

3. What is the kinetic energy of the ball at the highest point of the trajectory?

- (a) -2250 J (b) 2250 J (c) -4500 J (d) 4500 J

## Peer instruction questions

Suppose you have a 0.3 kg ball which you want to throw or drop. Under which of these conditions will the ball have the *greatest* kinetic energy just before it hits the ground? (Neglect any effects of air friction.)

- (a) You throw it with a velocity of 9m/s at angle of  $45^\circ$  **above** the horizontal at a distance of 1m above the ground.
- (b) You throw it with a velocity of 9m/s at angle of  $45^\circ$  **below** the horizontal at a distance of 1m above the ground.
- (c) You drop it from rest at a distance of 1m above the ground.
- (d) You drop it from rest at a distance of 2m above the ground

Examples of the work—kinetic energy relation:

Suppose  $F_{\text{total}} = \text{constant} = F_0$

$$W_{\text{total}} = K_f - K_i$$

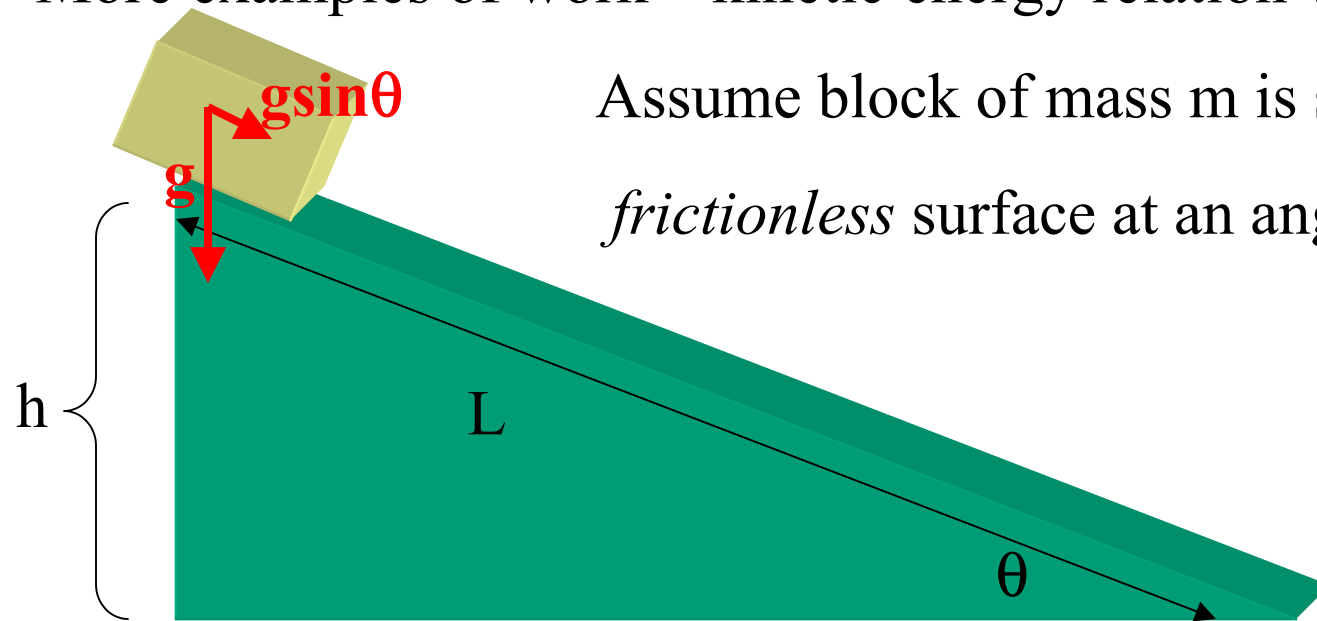
$$\rightarrow F_0(x_f - x_i) = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

In this case, we also know that  $F_0 = m a_0$  so that,

$$F_0(x_f - x_i) = m a_0 (x_f - x_i) = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$\rightarrow v_f^2 = v_i^2 + 2 a_0 (x_f - x_i)$$

More examples of work—kinetic energy relation without friction:

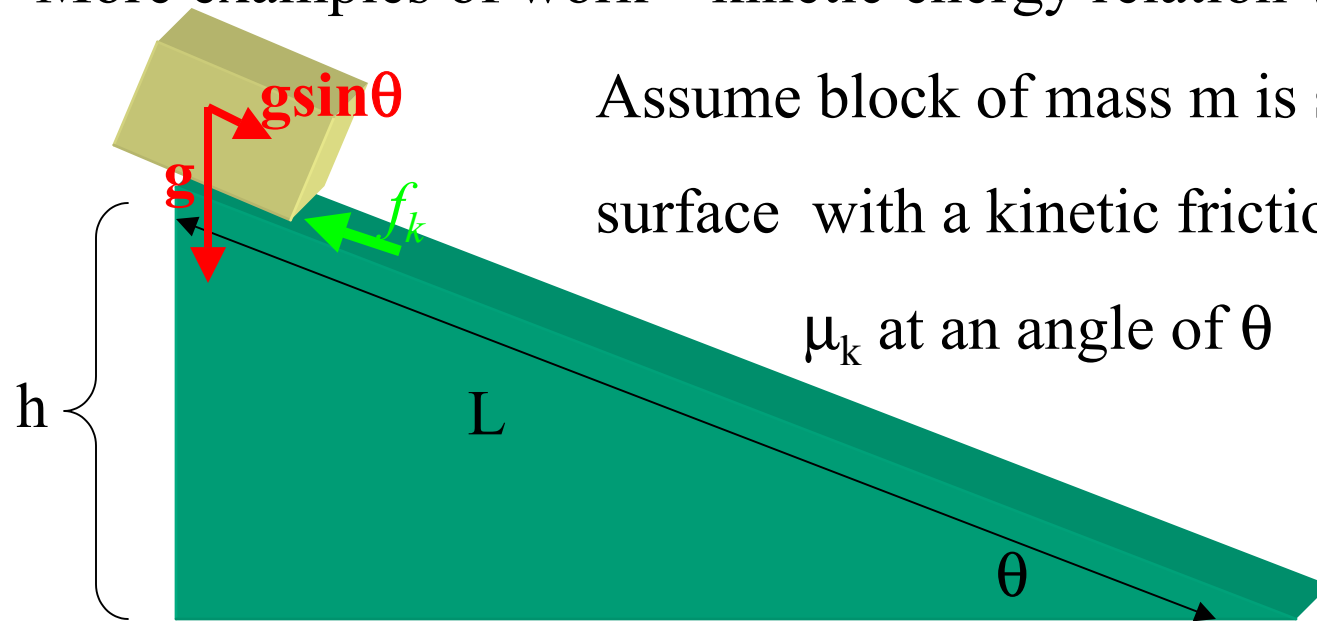


Assume block of mass  $m$  is sliding down a *frictionless* surface at an angle of  $\theta$

kinematic analysis:  $v_f^2 = v_i^2 + 2a_0(x_f - x_i) = 0 + 2(g \sin \theta)L = 2gh$

energy analysis:  $W_{\text{total}} = mgh = \frac{1}{2} m v_f^2$

More examples of work—kinetic energy relation with friction:



Assume block of mass  $m$  is sliding down a surface with a kinetic friction coefficient of  $\mu_k$  at an angle of  $\theta$

kinematic analysis:  $v_f^2 = v_i^2 + 2a_0(x_f - x_i) = 2(g \sin \theta - \mu_k g \cos \theta)L$

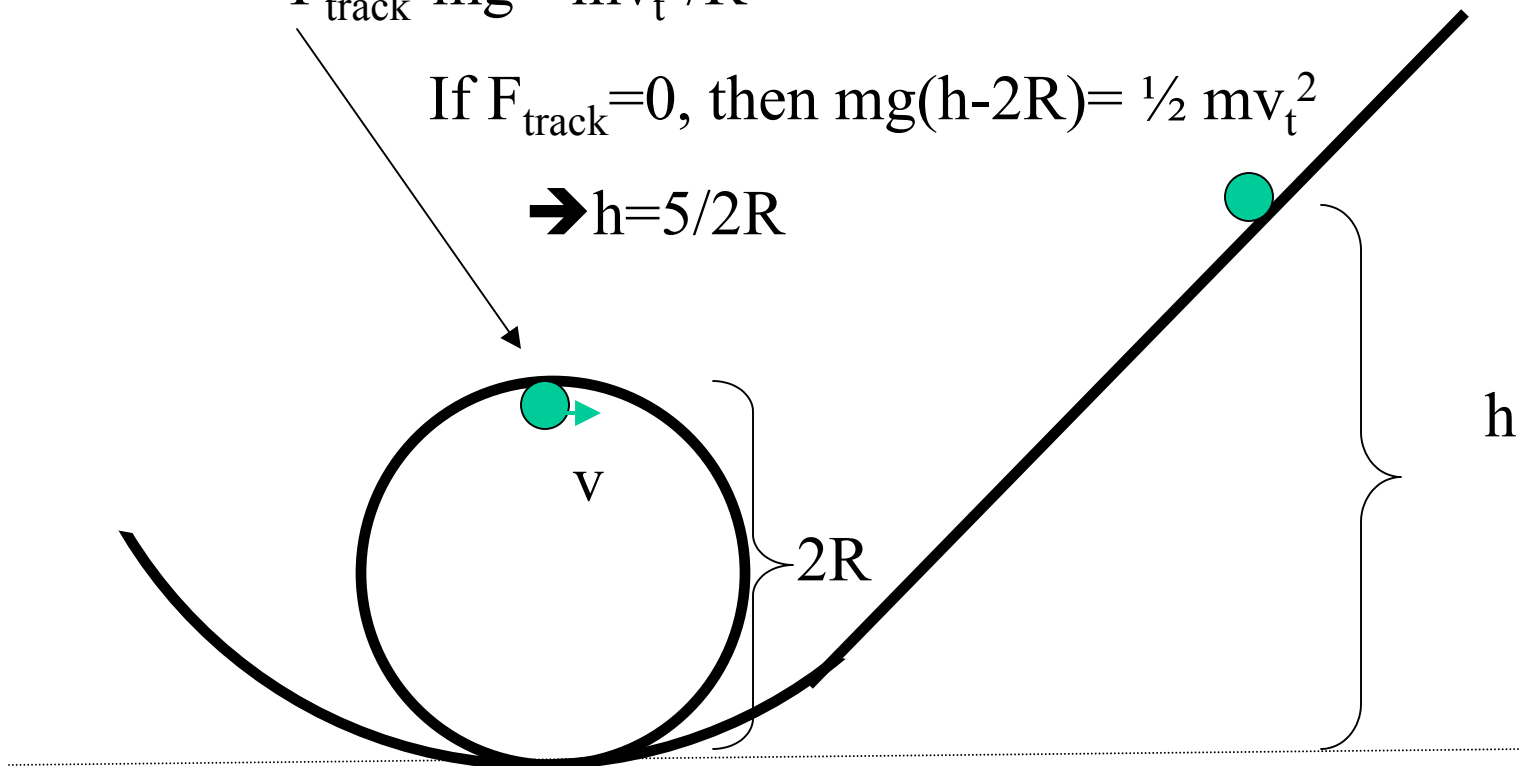
energy analysis:  $W_{\text{total}} = mgh - \mu_k mg \cos \theta L = \frac{1}{2} mv_f^2$

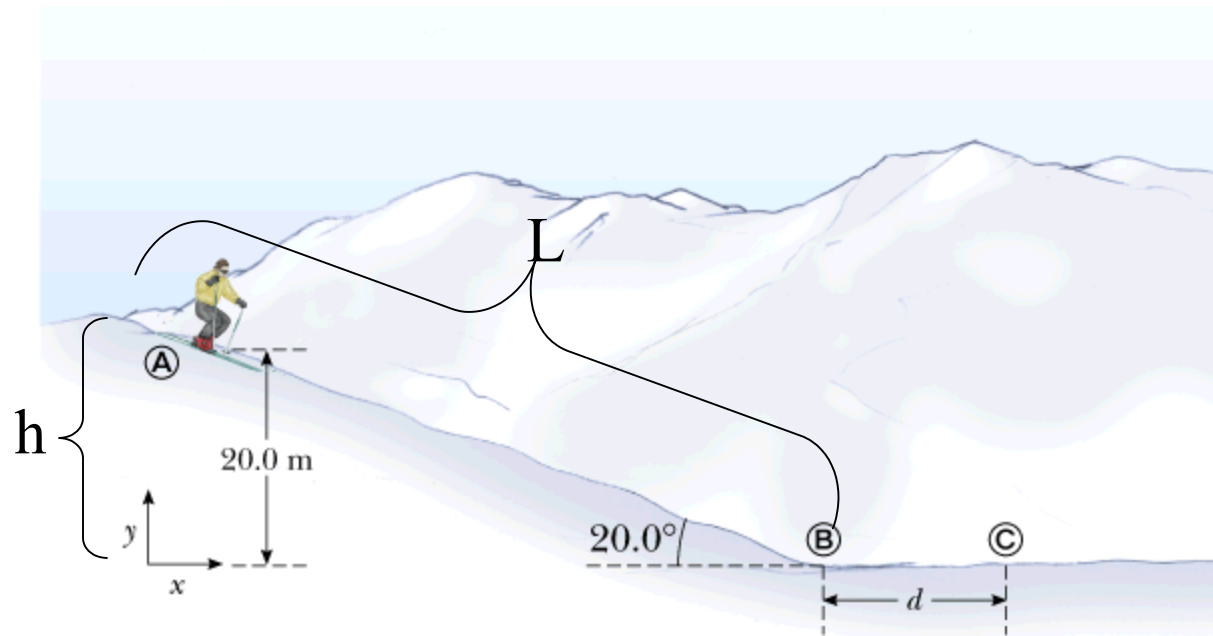


$$-F_{\text{track}} - mg = -mv_t^2/R$$

$$\text{If } F_{\text{track}} = 0, \text{ then } mg(h - 2R) = \frac{1}{2} mv_t^2$$

$$\Rightarrow h = 5/2 R$$



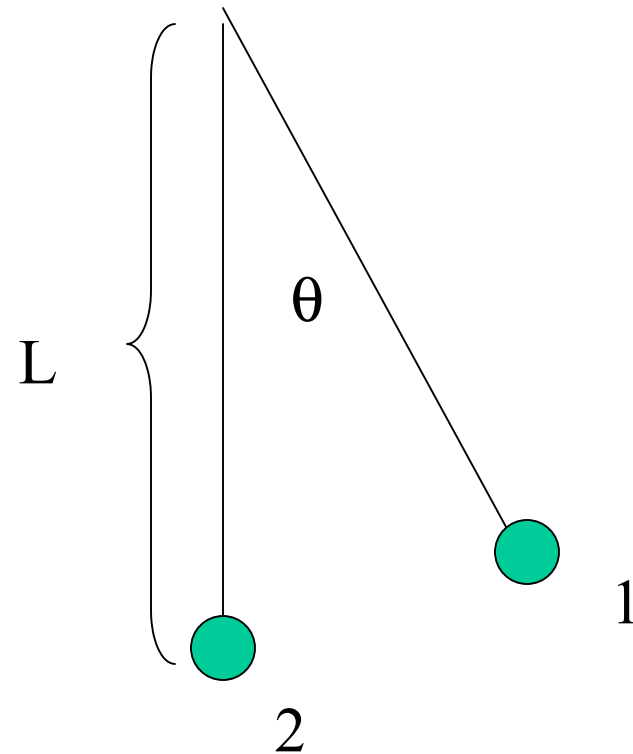


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Suppose that the coefficient of friction between the skis and the snow is  $\mu_k = 0.2$ . What is the stopping distance  $d$ ?

$$mgh - \mu_k mg \cos \theta L - \mu_k mgd = 0$$

$$d = h / \mu_k - L \cos \theta$$



A ball attached to a rope is initially at an angle  $\theta$ . After being released from rest, what is its velocity at the lowest point 2?

$$\sqrt{2gL(1 - \cos\theta)}$$