Announcements

1. Topics for today – torque, rotational motion
   – Chapters 10 & 11

2. Second hour exam Wed, Oct 16th

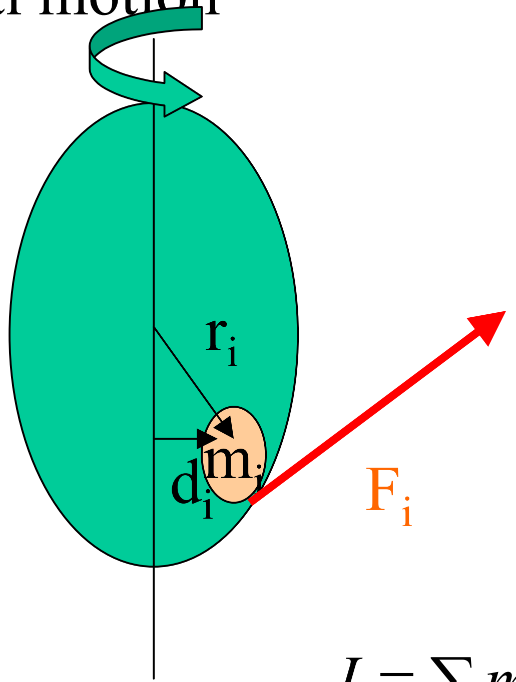
   practice exams available

Newton's second law applied to center-of-mass motion

$$\sum_i \mathbf{F}_i = \sum_i m_i \frac{d\mathbf{v}_i}{dt} \Rightarrow \mathbf{F}_{total} = M\frac{d\mathbf{v}_{CM}}{dt}$$

Newton's second law applied to rotational motion

$$\mathbf{F}_i = m_i \frac{d\mathbf{v}_i}{dt} \Rightarrow \mathbf{r}_i \times \mathbf{F}_i = \mathbf{r}_i \times m_i \frac{d\mathbf{v}_i}{dt}$$



$$\boldsymbol{\tau}_i = \mathbf{r}_i \times \mathbf{F}_i$$

$$\mathbf{v}_i = \boldsymbol{\omega} \times \mathbf{r}_i$$

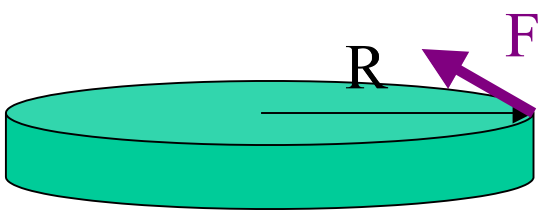
$$\Rightarrow \boldsymbol{\tau}_i = m_i \mathbf{r}_i \times \frac{d(\boldsymbol{\omega} \times \mathbf{r}_i)}{dt}$$

$$\Rightarrow \boldsymbol{\tau}_{total} = I\frac{d\boldsymbol{\omega}}{dt} = I\boldsymbol{\alpha}$$ (for rotating about principal axis)

$$I \equiv \sum_i m_i d_i^2$$

Homework problem #4

A horizontal 800 N merry-go-round is a solid disc of radius 1.50 m and is started from rest by a constant horizontal force of 50 N applied tangentially to the cylinder. Find the kinetic energy of solid cylinder after 3 s.

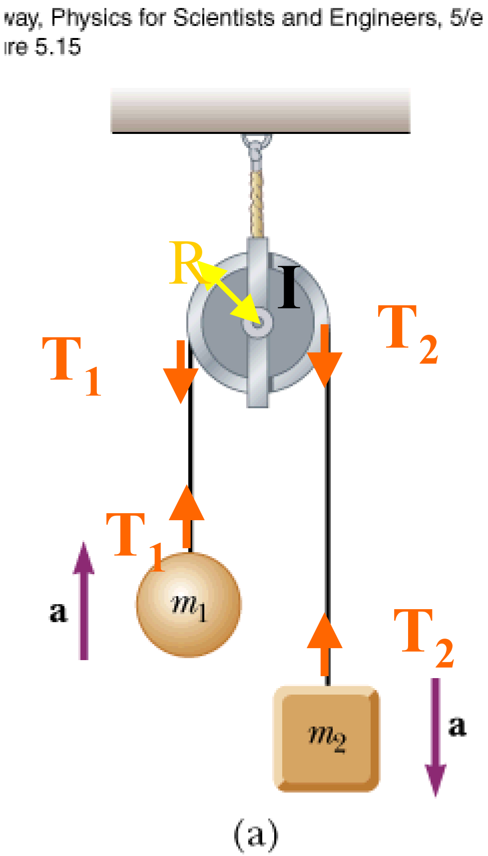


$K = \frac{1}{2} I \omega^2$ $\tau = I\alpha$ $\omega = \omega_i + \alpha t = \alpha t$

In this case $I = \frac{1}{2} m R^2$ and $\tau = FR$

$$K = g\frac{F^2}{mg}t^2 = 9.8\text{m/s}^2 \frac{(50N)^2}{800N}(3s)^2 = 275.625J$$

Re-examination of "Atwood's" machine



$$T_1 - m_1 g = m_1 a$$

$$T_2 - m_2 g = -m_2 a$$

$$\tau = T_2 R - T_1 R = I\,\alpha = I\,a/R$$

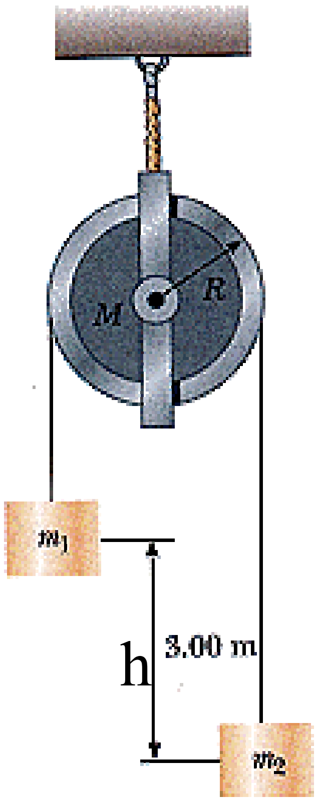
$$a = g\left(\frac{m_2 - m_1}{m_2 + m_1 + I/R^2}\right)$$

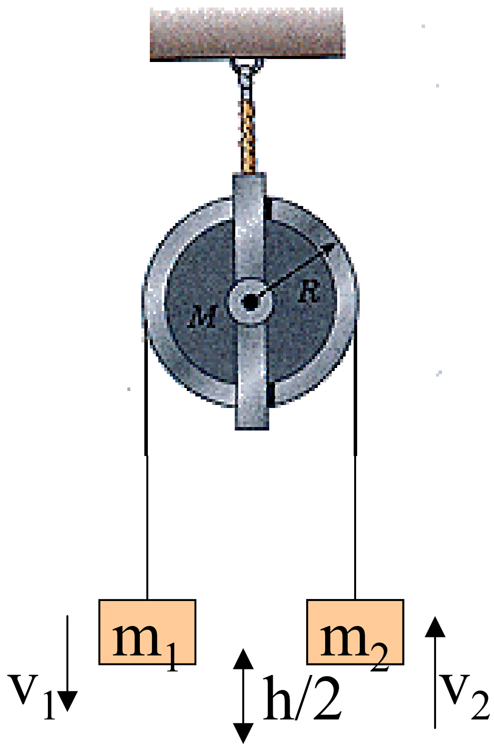
$$\tau = \frac{Ig}{R}\left(\frac{m_2 - m_1}{m_2 + m_1 + I/R^2}\right)$$

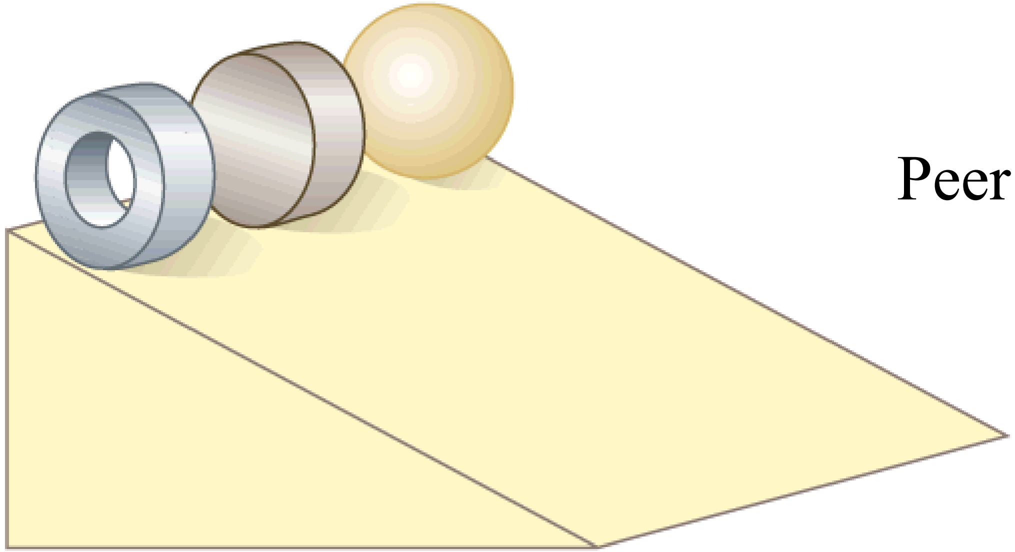
Homework problem:

Conservation of energy:

$$K_f + U_f = K_i + U_i$$







Peer instruction question

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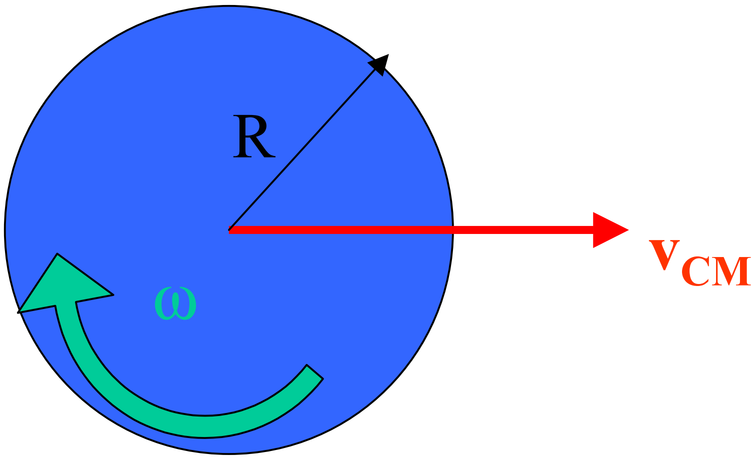
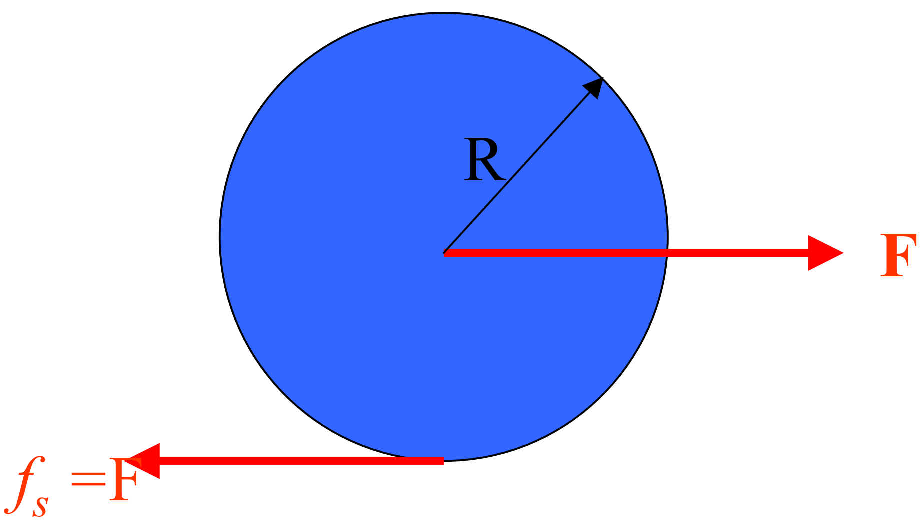
Three objects of uniform density – a solid sphere (a), a solid cylinder (b), and a hollow cylinder (c) -- are placed at the top of an incline. If they all are released from rest at the same elevation and roll without slipping, which object reaches the bottom first?

(a) solid sphere (b)solid cylinder (c)hollow cylinder

Rolling without slipping



$$\omega = v_{CM}/R$$



$$\tau = R\, f_s$$

Torque and angular momentum

Define angular momentum: $\mathbf{L} \equiv \mathbf{r} \times \mathbf{p}$

For an extended object: $L = I\omega$

Newton's law for torque:

$$\boldsymbol{\tau}_{total} = I\frac{d\boldsymbol{\omega}}{dt} = \frac{d\mathbf{L}}{dt}$$

➔ If $\boldsymbol{\tau}_{\text{total}} = 0$ then $\mathbf{L}$ = constant

In this case: $I_1\omega_1 = I_2\omega_2$