Announcements

1. Return hour exams (at end of lecture)

   Midterm grades were based on ~10 point scale of the average of your two exams.

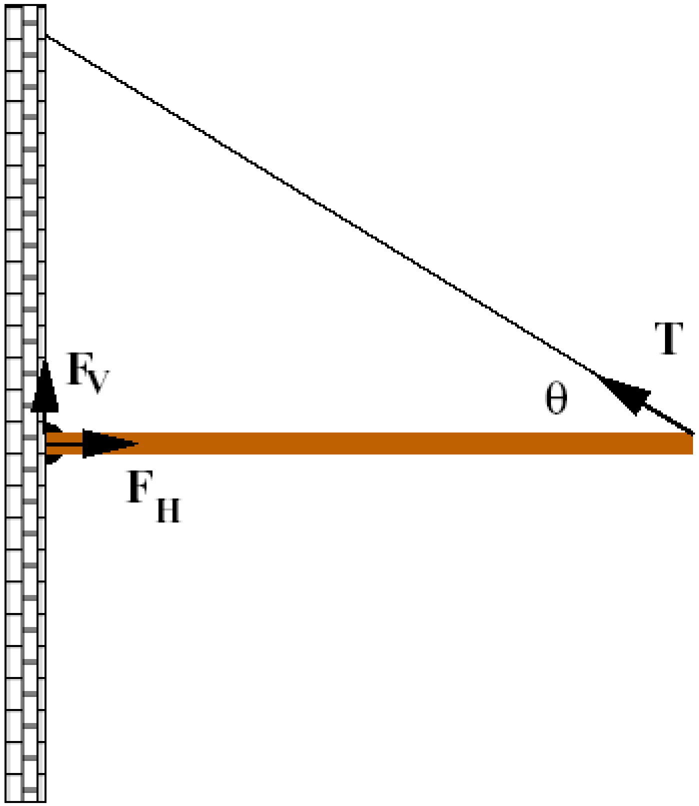
2. Physics lecture and workshop this week on teaching techniques for PHY 113 & 114 (http://www.wfu.edu/physics/seminars/chabay.html)

3. Today's lecture -- Chapter 13

   Hooke's law

   Simple harmonic motion

5.



The figure on the left shows a uniform plank of mass M=50 kg and length L=4 m. The moment of inertia of the plank for rotating about one end is I =266.67 kg $\cdot$ m^2. The plank is supported on the right by a massless rope with tension T at an angle $\theta = 30^o$ and on the left by a hinge which has a vertical support force F_V and and horizontal support force F_H.

(a) What is the tension T in the rope?

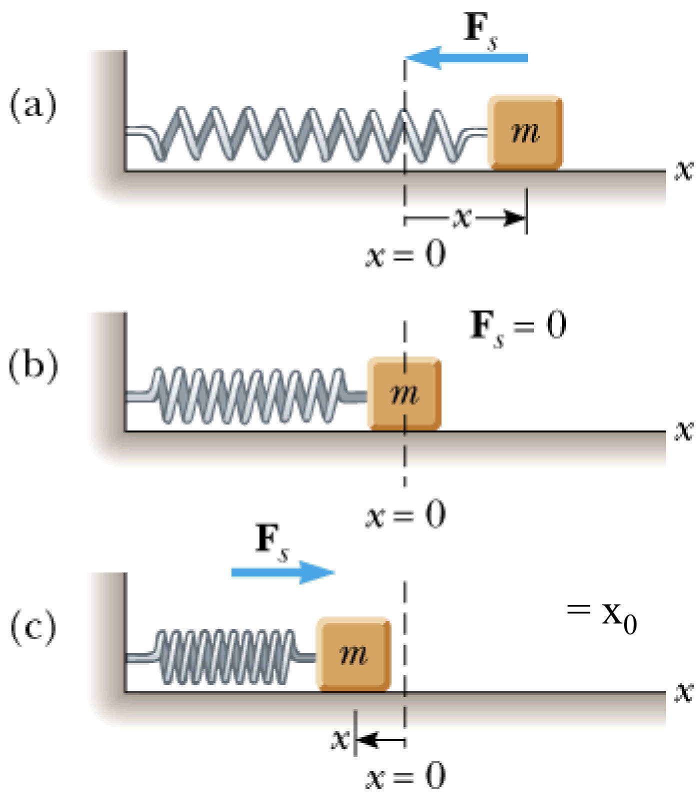
(b) Suppose that the rope is broken and the plank rotates about the hinge. At the instant that the rope breaks, what is the acceleration of the center of mass of the plank?

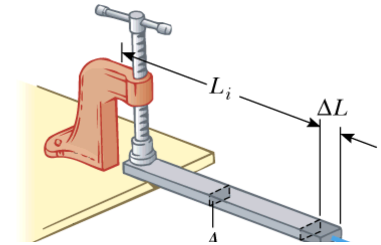
Behavior of materials:

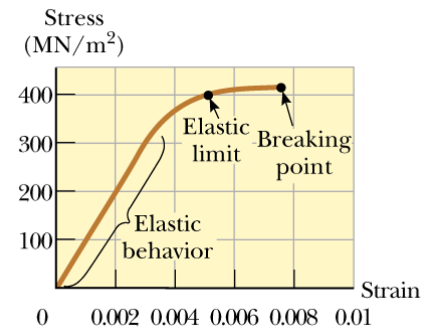
Hooke's law

$$F_s = -k(x-x_0)$$

Serway, Physics for Scientists and Engineers, 5/e
Figure 13.1







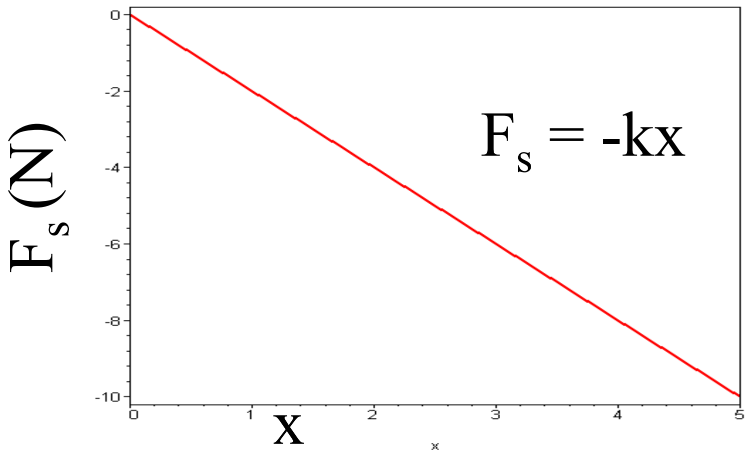
Young's modulus

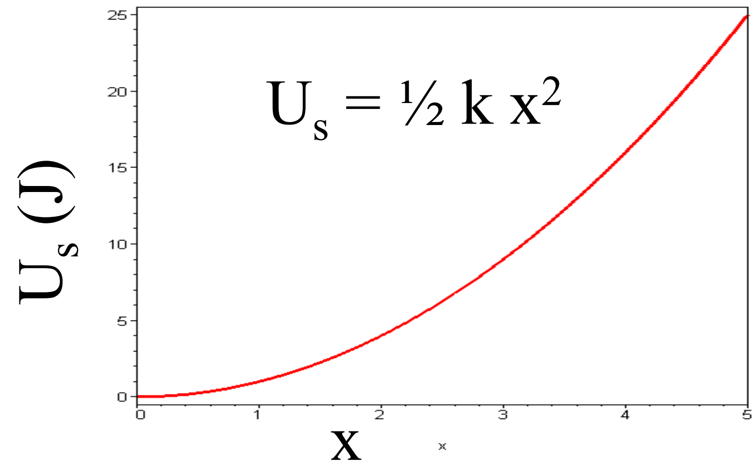
$$Y = \frac{F_{applied} / A}{\Delta L / L}$$

$$F_{material} = -F_{applied}$$

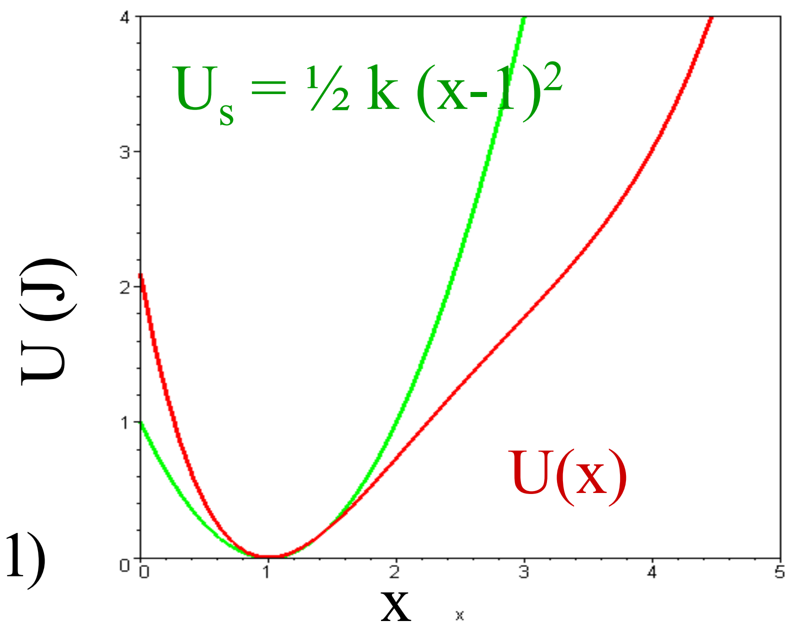
$$\Rightarrow F_{material} = -\left(\frac{YA}{L}\right)\Delta L$$

Potential energy associated with Hooke's law:





General potential energy curve:



$$k = \frac{d^2U}{dx^2}(x = 1)$$

Motion associated with Hooke's law forces

Newton's second law:

F = -k x = m a

$$F = -kx = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$ ➔ "second-order" linear differential equation

How to solve a second order linear differential equation:

Earlier example – constant force F_0 ➔ acceleration a_0

$$\frac{d^2x}{dt^2} = \frac{F_0}{m} \equiv a_0$$

$x(t) = x_0 + v_0 t + \frac{1}{2} a_0 t^2$

2 constants (initial values)

Hooke's law motion:

$$F = -kx = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

Forms of solution:

$$\left.\begin{aligned} x(t) &= A\cos(\omega t + \varphi) \\ x(t) &= A\cos(\omega t) + B\sin(\omega t) \end{aligned}\right\} \quad \text{where: } \omega \equiv \sqrt{\frac{k}{m}}$$

2 constants (initial values)

Verification: (Class exercise – write out steps of this “proof”)

Differential relations:

$$\frac{d\sin(\omega t+\varphi)}{dt}=\omega\cos(\omega t+\varphi)$$

$$\frac{d\cos(\omega t+\varphi)}{dt}=-\omega\sin(\omega t+\varphi)$$

Therefore: $$\frac{d^2A\cos(\omega t+\varphi)}{dt^2}=-\omega^2 A\cos(\omega t+\varphi)$$

$$\Rightarrow x(t)=A\cos(\omega t+\varphi) \qquad \textit{satisfies}$$

$$\frac{d^2x}{dt^2}=-\frac{k}{m}x \qquad \textit{provided} \qquad \textit{that} \qquad \omega^2=\frac{k}{m}$$

“Simple harmonic motion” in practice

A block with a mass of 0.2 kg is connected to a light spring for which the force constant is 5 N/m and is free to oscillate on a horizontal, frictionless surface. The block is displaced 0.05 m from equilibrium and released from rest. Find its subsequent motion.

$$\omega = \sqrt{k/m} = \sqrt{5/0.2}\,\text{rad/s} = 5\,\text{rad/s}$$

$x(t) = A\cos(\omega t+\phi)$ $\quad$ $x(0) = A\cos(\phi) = 0.05$ m

$v(t) = -A\omega\sin(\omega t+\phi)$ $\quad$ $v(0) = -A\omega\sin(\phi) = 0$ m/s

➔ $\phi = 0$ and $A = 0.05$ m

Peer instruction question:

A certain mass m on a spring oscillates with a characteristic frequency of 2 cycles per second. Which of the following changes to the mass would increase the frequency to 4 cycles per second?

(a) 2m (b) 4m (c) m/2 (d) m/4